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Original Research Article

Expression of Lorentz Transformation from Classical Mechanical Law

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Abstract

In this work derivation of Lorentz transformation by Newton second law using relativity speed. Also discussed many expression of relative mechanical relation and proper time and Relative coefficient form classical force and Minkowiski force in other side using Total relative energy of the moving particle to derivation Relative coefficient.
Keywords: Relative coefficient, relativity speed, total energy, derivation, momentum.

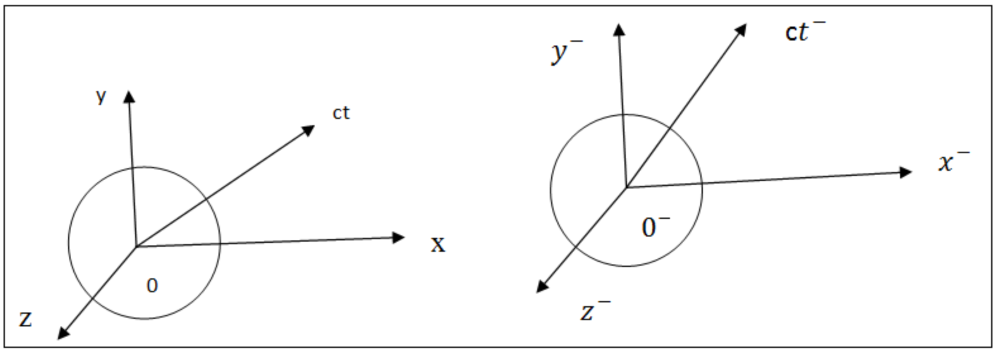
Introduction

The well-known Lorentz transformation, named after the Dutch physicist Hendrik Lorentz, is a set of equations relating the space and time coordinates of two inertial reference frames in relative uniform motion with respect to each other, so that coordinates can be transformed from one reference frame to another. Length contraction and time dilation are supposedly the principal outcome of the Lorentz transfor mation. Originally, Lorentz developed the transformation to explain, with other physicists (Larmor, Fitzgerald, and Point- caré), how the speed of light seemed to be independent of the reference frame, following the puzzling results of the famous Michelson-Morley experiment [1-3].

Four vector in relativity of space – time and the momentum – energy of particle are often expressed in four – vectors .they are defined so that the length of a four vectors is invariant under coordinate transformation [4, 5].

In this paper derivation of Lorentz transformation by Newton second law and expression of relativistic velocity by using classical force beside Lorentz force.

Consider two inertial frames of reference, (x, y, z, t) and (x^-, y^-, z^-, t^-) in translational relative motion with parallel corresponding axes [1]. Form the Lorentz transformation the basic idea is to derive a relationship between the space-time coordinates x, y, z, t as seen by observer (o) and the coordinates x^-, y^-, z^-, t^- seen by observer o^- Moving at a velocity u_x in this figure



Since the speed of light is the same (c) in the both figure. The liner transformation between (x, t) and x^-, t^- can be written as

$$x = \gamma(\bar{x} + v\bar{t}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\bar{x} = \gamma(x - vt) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$t = \gamma\left(\bar{t} + \frac{u\bar{x}}{c^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3)$$

$$\bar{t} = \gamma\left(t - \frac{ux}{c^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (4)$$

Relation between Force and Momentum

Form the Newton's second law the relation between force and momentum given by

$$F = \frac{dp}{dt} \ldots\ldots\ldots\ldots\ldots\ldots.. (5)$$

$$F.V = \frac{d\varepsilon}{dt} \ldots\ldots\ldots\ldots\ldots\ldots.. (6)$$

Where ε the total energy relativity of the moving particle also the momentum

$$p = mv$$

In the traditional mechanics the mass is not related to velocity

$$F = m_{\circ} \frac{dv}{dt} \ldots\ldots\ldots\ldots\ldots\ldots.. (7)$$

$$m_{\circ} = m\sqrt{1 - \frac{v^2}{c^2}}$$

In special relativity the for force is defined as the rote of change in the four momentum of a particle with respect to the particle proper time

First force from equation (5)

$$F = \frac{dp}{dt}$$

For a particle of constant invariant mass > 0 , p=mu

When $u = \gamma(c, u)$

$$F = ma = (\gamma\frac{f.u}{c}, \gamma f)$$

$$F = \frac{d}{dt}(\gamma mu) = \frac{dp}{dt}$$

And

$$f.u = \frac{d}{dt}(\gamma mc^2) = \frac{dE}{dt} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Multiplying both side in equation (5) by v we get

$$v.\frac{dp}{dt} = v.F \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots... (9)$$

The left side in equation (9) expression Hamilton equation

$$v = \frac{d\varepsilon}{dp} \quad and \quad q = \frac{dS}{dp}$$

$$S \equiv \text{Hamilton}$$

$$S = mc^2\sqrt{1 - \frac{\mathring{q}^2}{c^2}} \;, p = \frac{m\mathring{q}}{\sqrt{1 - \frac{\mathring{q}^2}{c^2}}}$$

$$q^{\circ} = \frac{pc^2}{\sqrt{mc^2 + p^2c^2}}$$

Form equation (9)

$$v.\frac{dp}{dt} = \frac{d\varepsilon}{dp}\frac{dp}{dt} = \frac{d\varepsilon}{dt} \ldots\ldots\ldots\ldots\ldots (10)$$

Form equation (9) and (10) we get

$$\frac{d\varepsilon}{dt} = F.v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (11)$$

In the classical mechanics the total energy equal the kinetic energy

$$E_t = T = \frac{1}{2}mv^2$$

The mass m change with velocity for S

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Also the mass $\bar{m}$ is not change with velocity for$\bar{S}$

$$\bar{F} = \bar{m}\frac{d\bar{v}}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Comparison between equation (12) and (13) the mass $\bar{m}$ must be related to velocity so that rewrite equation (13) again

$$\bar{F} = \bar{m}\frac{d\bar{v}}{dt} + \bar{v}\frac{d\bar{m}}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Form this figure the farm S moving by relativity speed U_x the equation (5) become [3]

$$\frac{dp_x}{dt} = F_x \rightarrow \frac{dp_y}{dt} = F_y \rightarrow \frac{dp_z}{dt} = F_z \ldots\ldots\ldots\ldots\ldots\ldots.. (15)$$

Also $\bar{S}$ moving by relative speed $\bar{u}_x$

$$\frac{d\bar{p}_x}{d\bar{t}} = \bar{F}_x \rightarrow \frac{d\bar{p}_y}{d\bar{t}} = \bar{F}_y \rightarrow \frac{d\bar{p}_z}{d\bar{t}} = \bar{F}_z \ldots\ldots\ldots\ldots\ldots\ldots.. (16)$$

Multiplying equation (11) to $-\frac{u}{c^2}$ we get

$$-\frac{u}{c^2}.\frac{d\varepsilon}{dt} = -\frac{u}{c^2}.F.v \ldots\ldots\ldots\ldots\ldots.. (17)$$

Addition equation (17) and (15) in the x- direction

$$\frac{dp_x}{dt} - \frac{u}{c^2}.\frac{d\varepsilon}{dt} = (F_x - \frac{u}{c^2}F.v) \ldots\ldots\ldots\ldots\ldots.. (18)$$

And the vector $\bar{F}.\bar{v}$ in the y, z direction

$$\frac{dp_x}{dt} - \frac{u}{c^2}.\frac{d\varepsilon}{dt} = F_x - \frac{u}{c^2}(F_y v_y + F_z v_z) \ldots\ldots\ldots\ldots\ldots.. (19)$$

Dividing equation (19) by $1 - \frac{uV_x}{c^2}$

$$\frac{d(p_x - \frac{u\varepsilon}{c^2})}{\mathrm{dt}(1 - \frac{uV_x}{c^2})} = F_x - \frac{\frac{u}{c^2}(F_y v_y + F_z v_z)}{1 - \frac{uV_x}{c^2}} \dots \dots \dots \dots \dots \dots (20)$$

Multiplying and dividing by (γ) left side in equation (20) and compare to equation (15) in x- direction

$$\bar{p}_x = \gamma\left(p_x - \frac{u\varepsilon}{c^2}\right) \dots \dots \dots \dots \dots (21)$$

$$d\bar{t} = \gamma \mathrm{dt}\left(1 - \frac{uv_x}{c^2}\right) \dots \dots \dots \dots \dots (22)$$

Similarly Lorentz transformation equation bring about the following expression

$$x^2 + y^2 + z^2 = t^2c^2 \dots \dots \dots \dots \dots \dots (23)$$

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 = \bar{t}^2c^2 \dots \dots \dots \dots \dots \dots (24)$$

Substitute equation (2) in equation (1) will lead to the following expression

$$x = \gamma[\gamma(x - vt) + v\bar{t}] \quad \rightarrow x = \gamma^2 x - \gamma^2 vt + \gamma v\bar{t}$$

$$\gamma v\bar{t} = x - \gamma^2 x + \gamma vt$$

dividing by (γv)

$$\bar{t} = \frac{x}{\gamma v}(1 - \gamma^2) + \gamma t \dots \dots \dots \dots \dots \dots \dots (25)$$

Derivation of the Lorentz transformation by Relative coefficient

$$1 - \gamma^2 = 1 - \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1 - \frac{v^2}{c^2} - 1}{1 - \frac{v^2}{c^2}} = \frac{-\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \dots \dots \dots \dots \dots \dots (26)$$

$$\frac{1 - \gamma^2}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} - \frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} = \frac{-\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = -\gamma\frac{v^2}{c^2} \dots \dots \dots \dots (27)$$

Form equation (25)

$$\bar{t} = \frac{x}{v}\frac{(1 - \gamma^2)}{\gamma} + \gamma t = \gamma\frac{v^2}{c^2}.\frac{x}{v} + \gamma t$$

Let v=u

$$\bar{t} = \gamma\left(t - \frac{ux}{c^2}\right) \dots \dots \dots \dots \dots \dots \dots (28)$$

The equation (28) all the same to equation (22). Subtracting equation (24) from equation (23) given that the y and z coordinates remain unaltered leads to [2]

$$x^2 - \bar{x}^2 = t^2c^2 - \bar{t}^2c^2 \dots \dots \dots \dots \dots \dots (29)$$

Rewrite equation (29) again leads to

$$x^2 = t^2c^2 \dots \dots \dots (30) \qquad \bar{x}^2 = \bar{t}^2c^2 \dots \dots \dots \dots \dots (31)$$

Indeed Lorentz transformation equation (2) and (4) can be lead to

$$\bar{x}^2 = \gamma^2(x^2 + v^2t^2 - 2xvt \dots \dots \dots \dots \dots \dots \dots \dots \dots (32)$$

$$\bar{t}^2c^2 = \gamma^2(x^2t^2 + \frac{v^2x^2}{c^2} - 2xvt \dots \dots \dots \dots \dots \dots \dots \dots (33)$$

Eliminating the tem 2xvt form equation (32) and equation (33) yields

$$x^2 + v^2t^2 - \frac{\bar{x}^2}{\gamma^2} = t^2c^2 + \frac{v^2x^2}{c^2} - \frac{\bar{t}^2c^2}{\gamma^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Similarly Lorentz transformation equation (1) and (3) can be expression

$$-\bar{x}^2 - v^2\bar{t}^2 + \frac{x^2}{\gamma^2} = -\bar{t}^2c^2 - \frac{v^2\bar{x}^2}{c^2} + \frac{t^2c^2}{\gamma^2} \ldots\ldots\ldots\ldots\ldots (36)$$

Adding equation (35) and equation (36) we get

$$x^2\left[1 + \frac{1}{\gamma^2}\right] - \bar{x}^2\left[1 + \frac{1}{\gamma^2}\right] + v^2(t^2 - \bar{t}^2) = t^2c^2\left(1 + \frac{1}{\gamma^2}\right) - \bar{t}^2c^2\left[1 + \frac{1}{\gamma^2}\right] + \frac{v^2}{c^2}(x^2 - \bar{x}^2)$$

can be written

$$(x^2 - \bar{x}^2)\left(1 + \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right) = c^2(t^2 - \bar{t}^2)(1 + \frac{1}{\gamma^2} - \frac{v^2}{c^2})$$

$$x^2 - \bar{x}^2 = c^2(t^2 - \bar{t}^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (37)$$

Rewrite equation (37) again leads to

$$x^2 = t^2c^2 \ldots\ldots\ldots. (38) \qquad \bar{x}^2 = \bar{t}^2c^2 \ldots\ldots\ldots\ldots\ldots.. (39)$$

Derivation of the Lorentz transformation by Lorentz force

form equation (15) in the y- direction $\frac{dp_y}{dt} = F_y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (40)$

And minkowski force $\vec{F}_m = \frac{\vec{F}}{\sqrt{1-\frac{v^2}{c^2}}}$

Also the force in $\bar{O}$ point at y – direction

$$\vec{F}_y = \frac{F_y}{\gamma\sqrt{1 - \frac{v^2.}{c^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (41)$$

And Lorentz force in O point at y- direction

$$F_y = \frac{\vec{F}_y}{\gamma\sqrt{1 - \frac{v^2.}{c^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (42)$$

Substitute equation (41) in equation (42) we get

$$\gamma\left(1 - \frac{uV_x}{c^2}\right).\left(1 + \frac{u\bar{V}_x}{c^2}\right)\gamma = 1$$

$$\left(1 - \frac{uV_x}{c^2}\right).\left(1 + \frac{u\bar{V}_x}{c^2}\right) = \frac{1}{\gamma^2}$$

$$\left(1 - \frac{uV_x}{c^2}\right).\left(1 + \frac{u\bar{V}_x}{c^2}\right) = 1 - \frac{u^2}{c^2} = \frac{1}{\gamma^2}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

form this figure $r = ct$ and $\bar{r} = c\bar{t}$ form equation (1) and (2)

$$x = \gamma(\gamma(x - vt) + v\bar{t}$$

$$\bar{t} = \gamma t + \frac{(1-\gamma^2)}{\gamma v}.x \ldots\ldots\ldots\ldots\ldots\ldots..(44)$$

Replacing $\bar{x}, \bar{y}, \bar{z},$ and $\bar{t}$ in equation (24)

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 = \bar{t}^2 c^2$$

$$\gamma^2(x - vt) + y^2 + z^2 = c^2\left[\gamma t + \frac{(1-\gamma^2)}{\gamma v}x\right]^2$$

$$\gamma^2 x + \gamma^2 v^2 t^2 - 2\gamma^2 vtx + y^2 + z^2 = c^2\gamma^2 t^2 + \frac{(1-\gamma^2)^2}{\gamma^2 v^2}c^2 x^2 + 2\frac{(1-\gamma^2)}{v}txc^2$$

$$\left[\gamma^2 - \frac{(1-\gamma^2)^2 c^2}{\gamma^2 v^2}\right]x^2 - 2v\gamma^2 tx + y^2 + z^2 = (c^2\gamma^2 - v^2\gamma^2)t^2 + 2\frac{(1-\gamma^2)}{v}txc^2$$

$$\left[\gamma^2 - \frac{(1-\gamma^2)^2}{\gamma^2 v^2}c^2\right]x^2 - \left[2v\gamma^2 + 2\frac{(1-\gamma^2)}{v}c^2\right]tx + y^2 + z^2 = (c^2\gamma^2 - v^2\gamma^2)t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45)$$

Comparing the coefficient of t^2 in equation (23) and coefficient of t^2 in equation (45)

$$c^2\gamma^2 - v^2\gamma^2 = c^2 \quad \rightarrow \gamma^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(46)$$

Discussion

In this discussion they are many expression of Lorentz transformation to derivation a relationship between space time coordinates. In the first one expression of relationship between Newton Second Law and Energy in classical mechanics showing proper time in equation (22) also this equation showing in equation (28) by using relativity speed equation and Hamilton equation. In other side derivation Relativity coefficient by using minkowski force and Lorentz force showing in equation (43) and equation (46)) the Relativity coefficient in equation (43) to similar the Relativity coefficient in equation (46). Derivation of the Lorentz transformation by Relative coefficient in equation (30) and equation (31) similarly to result in equation (38) and equation (39).

Conclusion

They are new derivation of Lorentz transformation by using classical mechanics force minkowski force. Also many expression of proper time by using relativity speed also new definition of Relativity coefficient.

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