DOI: 10.21276/sjbms.2017.2.2.5

Saudi Journal of Business and Management Studies
Scholars Middle East Publishers
Dubai, United Arab Emirates
Website: http://scholarsmepub.com/

ISSN 2415-6663 (Print)
ISSN 2415-6671 (Online)

Linear Programming Problems in Fuzzy Decision Space

Padmabati Gahan[1], Monalisha Pattnaik[1], Sourav Dhal[2]
[1]Dept. of Business Administration, Sambalpur University, Jyotivihar, Sambalpur, India-768019
[2]Cluster Supply Planner, Hindustan Coca Cola Beverage Pvt. Ltd., Bengaluru, Karnataka, India

***Corresponding Author:**
Monalisha Pattnaik
Email: monalisha_1977@yahoo.com

Abstract: In this paper the optimal solution for linear programming is derived where some parameters are fuzzy in numbers. In practice, there are many problems arise if the decision parameters are crisp in nature, and such problems are usually solved by introducing either probabilistic programming or multi objective programming methods. Unfortunately all these methods have shortcomings. In this note, the concept of fuzzy numbers is introduced, which is a very effective method for solving these problems. With the problem assumptions, the optimal solution can still be theoretically solved using the simplex based method. Fuzzy decision variables can be initially generated and then solved and improved sequentially using the fuzzy decision approach by introducing robust ranking technique. The proposed procedure was programmed through MATLAB (R2009a) version software for representing four dimensional slice diagrams to its application. The model is illustrated with an application which incorporates all concepts of a fuzzy arithmetic approach to draw managerial insights.
Keywords: Linear programming, Fuzzy, Optimal, Robust ranking technique

INTRODUCTION

Linear programming is an optimization technique and is applied in real-world problems most frequently. It is important to introduce new tools in the approach that allow the model to fit into the real world as much as possible. Any linear programming model representing real-world situations involves a lot of parameters whose values are assigned by experts, and in the conventional approach, they are required to fix an exact value to the aforementioned parameters. However, both experts and the decision maker frequently do not know precisely the value of the parameters. If the exact values are suggested then these are only for statistical inference which is derived from the past data and their stability is doubtful, so the parameters of the problem are usually defined by the decision maker in a uncertain space. Therefore, it is useful to consider the knowledge of the experts about the parameters which is known as fuzzy data. Two significant questions may be found in these kinds of problems:

- How to handle the relationship between the fuzzy parameters?
- How to find the optimal values for the fuzzy multi-objective function?

The answer of the above questions is related to the problem of ranking fuzzy numbers. In fuzzy decision making problems, the concept of maximizing decision was introduced by Bellman and Zadeh [1]. Zimmerman [2] presented a fuzzy approach to multi-objective linear programming problems in his classical paper. Lai and Hwang [3] considered the situations where all parameters are in fuzzy. Lai and Huang [4] assume that the parameters have a triangular possibility distribution. Gani *et al*. [5] introduce fuzzy linear programming problem based on L-R fuzzy number. Jimenez *et al*. [6] propose a method for solving linear programming problems where all coefficients are, in general, fuzzy numbers and using linear ranking technique. Bazaar *et al*. [7] and Nasseri *et al*. [8] define linear programming problems with fuzzy numbers and simplex method is used for finding the optimal solution of the fuzzy problem. Rangarajan and Solairaju [9] compute improved fuzzy optimal Hungarian assignment problems with fuzzy numbers by applying Robust ranking techniques to transform the fuzzy assignment problem to a crisp one. Pattnaik [10] presented a fuzzy approach to several linear and nonlinear inventory models. Swarup *et al*. [11] explain the method to obtain sensitivity analysis or post optimality analysis of the different parameters in the linear programming problems.

Looking at the property of representing the preference relationship in fuzzy terms, ranking methods can be classified into two approaches. One of them associates, by means of different functions, each fuzzy number to a single of the real line and then a total crisp order relationship between fuzzy numbers is established. The other approach ranks fuzzy numbers by means of a fuzzy relationship. It allows decision maker to present his preference in a gradual way, which in a linear programming problem allows it to be handled with different degrees of satisfaction of constraints. This paper considers fuzzy multi-objective linear programming problems whose parameters are fuzzy numbers but the decision variables are in crisp space. The aim of this paper is to introduce Robust ranking technique for defuzzifying the fuzzy parameters and then sensitivity analysis for requirement vector in the constraint function is also performed that permits the interactive participation of decision maker in all steps of decision process, expressing his opinions in linguistic terms. The major techniques used in the above research articles are summarized in Table 1.

Table-1: Major Characteristics of Fuzzy Linear Programming Models on Selected Researches

Author(s) and Published Year	Structure of the Model	Fuzzy Number	Objective Model	Model Type	Ranking Function
Zimmermann [2]	Fuzzy	Triangular	Single	Cost	Linear
Maleki *et al.* [13]	Fuzzy	Trapezoidal	Multi	Profit	Linear
Jimenez *et al.* [6]	Fuzzy	Triangular	Multi	Cost	Linear
Nasseri et al. [8]	Fuzzy	Trapezoidal	Multi	Profit	Linear
Present Paper (2017)	Fuzzy	Trapezoidal	Multi	Profit	Robust

The remainder of this paper is organized as follows. In Section 2, fuzzy numbers are introduced and some of the results are obtained by applying fuzzy arithmetic on them. Assumptions, notations and definitions are provided for the development of the model. In Section 3, robust ranking technique is introduced for solving fuzzy number linear programming problems. In Section 4, a linear programming problem is proposed with fuzzy variables and in Section 5 a fuzzy version of the simplex algorithm is explained for solving this problem. An application is presented to illustrate the development of the model in Section 6. Finally Section 7 deals with the summary and the concluding remarks.

Preliminaries

The fundamental notation of fuzzy set theory is reviewed and initiated by Bellman and Zadeh [1]. Definitions taken from Zimmerman [2] are given below.

Definition 2.1 *Fuzzy sets*

If X is a collection of objects denoted generally by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x\epsilon X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set$\tilde{A}$. The membership function maps each element of X to a membership value between 0 and 1.

Definition 2.2 *Support of a fuzzy set*

The support of a fuzzy set $\tilde{A}$ is the set of all points x in X such that $\mu_{\tilde{A}}(x) > 0$. That is $support\ (\tilde{A}) = \{x/\mu_{\tilde{A}}(x) > 0\}$.

Definition 2.3 $\alpha -$ *level of fuzzy set*

The $\alpha -$ cut (or) $\alpha -$ level set of a fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. That is $\tilde{A}_{\alpha} = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 2.4 *Convex fuzzy set*

A fuzzy set $\tilde{A}$ is convex if, $\mu_{\tilde{A}}\big(\lambda x_1 + (1-\lambda x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big), x_1, x_2 \epsilon X$ and $\lambda\epsilon[0,1]$. Alternatively, a fuzzy set is convex, if all $\alpha -$level sets are convex.

Definition 2.5 *Convex normalized fuzzy set*

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that it exists at least one $x_0 \epsilon R$ with $\mu_{\tilde{A}}(x_0) = 1$ and $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.6 *Trapezoidal fuzzy numbers*

Among the various fuzzy numbers, triangular and trapezoidal fuzzy numbers are of the most important. Note that, in this study trapezoidal fuzzy numbers are only considered. A fuzzy number is a trapezoidal fuzzy number if the membership function of its be in the following function of it being in the following form:

Any trapezoidal fuzzy number by $\tilde{a} = (a^L, a^U, \alpha, \beta)$, where the support of $\tilde{a}$ is $(a^L - \alpha, a^U + \beta)$ and the modal set of $\tilde{a}$ is $[a^L, a^U]$. Let *F(R)* is the set of trapezoidal fuzzy numbers.

Definition 2.7 *Arithmetic on fuzzy numbers*

Let $\widetilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x\epsilon R$. Then, the results of applying fuzzy arithmetic on the trapezoidal fuzzy numbers as shown in the following:

Image of $\widetilde{a}: -\widetilde{a} = (-a^U, -a^L, \beta, \alpha)$

Addition: $\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$

Scalar Multiplication: $x > 0,\ x\widetilde{a} = (xa^L, xa^U, x\alpha, x\beta)$ and $x < 0,\ x\widetilde{a} = (xa^U, xa^L, -x\alpha, -x\beta)$

Ranking Function

A convenient method for comparing of the fuzzy numbers is by use of ranking functions. A ranking function is a map from *F(R)* into the real line. The orders on *F(R)* are:

$\tilde{a} \geq \tilde{b}$ if and only if $\Re(\tilde{a}) \geq \Re(\tilde{b})$
$\tilde{a} > \tilde{b}$ if and only if $\Re(\tilde{a}) > \Re(\tilde{b})$
$\tilde{a} = \tilde{b}$ if and only if $\Re(\tilde{a}) = \Re(\tilde{b})$

Where, $\tilde{a}$ *and* $\tilde{b}$ are in *F (R)*. It is obvious that $\tilde{a} \leq \tilde{b}$ if and only if $\tilde{b} \geq \tilde{a}$. Since there are many ranking function for comparing fuzzy numbers but robust ranking function is applied. Robust's ranking technique satisfies compensation, linearity and additive properties and provides results which are consistent with human intuition. Give a convex fuzzy number $\tilde{a}$, the Robust's Ranking index is defined by $\Re(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha$, where (a^L, a^U) is the α-level cut of the fuzzy number $\tilde{a}$.

In this paper this method for ranking the objective values. The Robust ranking index $\Re$ $(\tilde{a})$ gives the representative value of the fuzzy number $\tilde{a}$. It satisfies the linearity and additive property.

Fuzzy Linear Programming Problems

However, when formulating a mathematical programming problem which closely describes and represents a real-world decision situation, various factors of the real world system should be reflected in the description of objective functions and constraints involve many parameters whose possible values may assigned by experts. In the conventional approaches, such parameters are required to be fixed at some values in an experimental and subjective manner through the experts' understanding of the nature of the parameters in the problem-formulation process.

It must be observed that, in most real-world situations, the possible values of these parameters are often only imprecisely known to the experts. With this observation in mind, it would be certainly more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers.

Definition 4.1 *Linear programming*

A linear programming (LP) problem is defined as:
$Max\ z = cx$
$s.t.\ Ax \leq b\ or\ Ax \geq b\ or\ Ax = b$
$\forall x \geq 0$
Where, $c = (c_1, c_2, \dots \dots .. c_n)$, $b = (b_1, b_2, \dots \dots b_m)^T$, and $A = [a_{ij}]_{m\times n}$.
In the above problem, all of the parameters are crisp. Now, if some of the parameters be fuzzy numbers then fuzzy linear programming is obtained which is defined in the next section.

Definition 4.2 *Fuzzy linear programming*

Suppose that in the linear programming problem some parameters be fuzzy numbers. Hence, it is possible that some coefficients of the problem in the objective function, technical coefficients the right hand side coefficients or decision making variables be fuzzy number Maleki [12], Maleki *et al*. [13], Rommelfanger *et al*. [14] and Verdegay [15]. Here, the linear programming problems with fuzzy numbers in the objective function.

Definition 4.3 *Fuzzy number linear programming*

A fuzzy number linear programming (FNLP) problem is defined as follows:
$Max\ \tilde{z} = \tilde{c}x$
$s.t.\ Ax = b$
$x \geq 0$
Where, $b \epsilon R^m$, $x \epsilon R^n$, $A \epsilon R^{m\times n}$, $\tilde{c}^T \epsilon ((F(R))^n$, and $\Re$ is a Robust ranking function.

Definition 4.4 *Fuzzy feasible solution*

The vector $x \epsilon R^n$ is a feasible solution to FNLP if and only if x satisfies the constraints of the problem.

Definition 4.5 *Fuzzy optimal solution*

A feasible solution x^* is an optimal solution for FNLP, if for all feasible solution x for FNLP, then $\tilde{c}x^* \geq \tilde{c}x$.

Definition 4.6 *Fuzzy basic feasible solution*

The basic feasible solution for FNLP problems is defined as: Consider the system $Ax = b$ and $x \geq 0$, where A is an $m \times n$ matrix and b is an m vector. Now, suppose that $rank(A, b) = rank(A) = m$. Partition after possibly rearranging the columns of A as $[B, N]$ where B, $m \times m$, is nonsingular. It is obvious that $rank(B) = m$. The point $x = (x_B^T, x_N^T)^T$ where, $x_B = B^{-1}b$, $x_N = 0$ is called a basic solution of the system. If $x_B \geq 0$, then x is called a basic feasible solution (BFS) of the system. Here *B* is called the basic matrix and *N* is called the non basic matrix.

A Fuzzy Version of Simplex Algorithm

For the solution of any FNLP by Simplex algorithm, the existence of an initial basic feasible fuzzy solution is always assumed. The steps for the computation of an optimum fuzzy solution are as follows:
Step-1 Check whether the objective functions of the given FNLP is to be maximized or minimized. If it is to be minimized then converting it into a problem of maximizing it by using the result $Minimum\ \tilde{z} = -Maximum(-\tilde{z})$
Step-2 Check whether all $\tilde{b}_i (i = 1,2, \dots .m)$ are non-negative. If any one $\tilde{b}_i$ is negative then multiply the corresponding inequation of the constant by -1, so as to get all $\tilde{b}_i$ $(i = 1,2, \dots .m)$ non-negative.
Step-3 Convert all the inequations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the cost of these variables equal to zero.

Step-4 Obtain an initial basic feasible solution to the problem in the problem in the form of $\tilde{x}_B = \tilde{B}^{-1}\tilde{b}$ and put in the first column of the simplex table.

Step-5 Compute the net evaluations $\tilde{\Delta}_j = \tilde{z}_j - \tilde{c}_j$ $(j = 1,2, \dots .n)$ by using the relation $\tilde{\Delta}_j = \tilde{c}_B \tilde{y}_j - c_j$. Examine the sign $\tilde{\Delta}_j$.

i) If all $\tilde{\Delta}_j \geq \tilde{0}$ then the initial basic feasible fuzzy solution $\tilde{x}_B$ is an optimum basic feasible fuzzy solution.

ii) If at least one $\tilde{\Delta}_j < \tilde{0}$, proceed on to the next step.

Step-6 If there are more than one negative $\tilde{\Delta}_j$, then choose the most negative of them. Let it be $\tilde{\Delta}_r$ for some *j=r*.

i) If all $\tilde{y}_{ir} \leq 0, (i = 1,2, \dots m)$, then there is an unbounded solution to the given problem.

ii) If at least one $\tilde{y}_{ir} > 0, (i = 1,2, \dots m)$, then the corresponding vector $\tilde{y}_r$ enter the basis $\tilde{y}_B$.

Step-7 Compute the $\frac{\tilde{x}_{Bi}}{\tilde{y}_{ir}}, i = 1,2, \dots .m$ and choose minimum of them. Let minimum of these ratios be $\tilde{x}_{Br}/\tilde{y}_{kr}$. Then the vector $\tilde{y}_k$ will level the basis $\tilde{y}_B$. The common element $\tilde{y}_{kr}$, which in the k[th] row and r[th] column is known as leading number of the table.

Step-8 Convert the leading number to unit number by dividing its row by the leading number itself and all other number itself and all other elements in its column to zero.

$$\tilde{\boldsymbol{y}} \approx \tilde{y}_{ij} - \left(\frac{\tilde{y}_{kj}}{\tilde{y}_{kr}}\right)\tilde{y}_{ir}, i = 1,2, \dots .m+1; i \neq k \text{ and } \tilde{\boldsymbol{y}}_{\boldsymbol{kj}}$$
$$\approx \left(\frac{\tilde{y}_{kj}}{\tilde{y}_{kr}}\right), j = 0,1,2, \dots n$$

Step-9 Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or this is an indication of an unbounded solution.

Application

In this section the application of fuzzy version simplex algorithm solution to FLPP has been presented. This application is the standard product mix problem.

6.1 Product Mix Problem

A company produces three products P_1, P_2 *and* P_3 each of which must be processed through three departments D_1, D_2 *and* D_3. The approximate time, in hours, each P_i spends in each D_j is given in Table 2.

Each department has only so much time available each week. These times are given in following numbers which are estimates of the maximum time available per week, in hours, for each department: (1) for D_1 288 h; (2) 312 h for D_2; and (3) D_3 has 124 h. finally the selling price for each product can vary a little due to small discounts to certain customers but the following average selling prices are: (1) \$6 for P_1; (2) \$8 for P_2 and for P_3 \$6/unit. The company wants to determine the number of units to produce for each product per week to maximize its revenue.

Table-2: Times of product P_i is in department D_j

Product	Department		
	D_1	D_2	D_3
P_1	6	8	3
P_2	12	8	6
P_3	2	4	1

Since all selling price numbers given are uncertain, the FLPP model is formulated. The Trapezoidal fuzzy number for each value given is obtained. So, the FLPP is given by:

Max $\tilde{z} = (4,6,7,8)x_1 + (5,8,9,10)x_2 + (5,6,8,9)x_3$

Such that

$6x_1 + 8x_2 + 3x_3 \leq 288$

$12x_1 + 8x_2 + 6x_3 \leq 312$

$2x_1 + 4x_2 + x_3 \leq 124$

$\forall\, x_1, x_2, x_3 \geq 0$

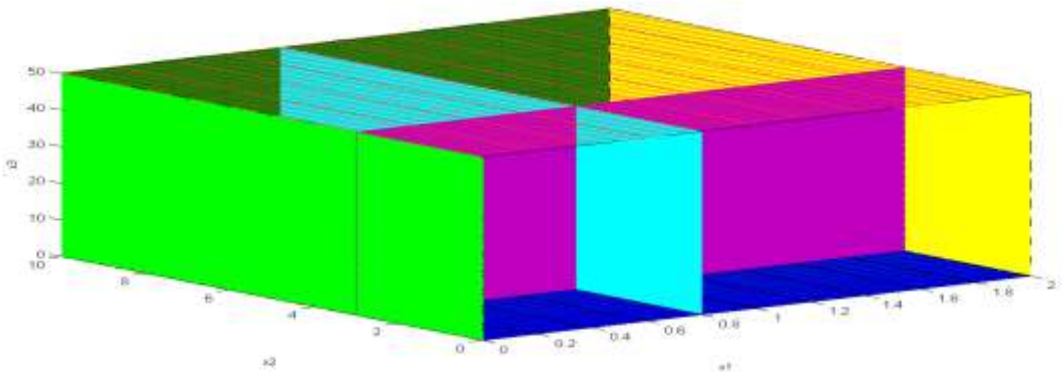

Fig-1: 4- Dimensional Slice and Mesh plot of Fuzzy Total Profit $z(x_1, x_2, x_3)$, x_1, x_2 *and* x_3.

From Table 3 it is found that the fuzzy optimal solutions are $\tilde{x}_1 = 0, \tilde{x}_2 = 3, \tilde{x}_3 = 48$ *and* $\tilde{z} = 360$. Fig. 1 shows the four dimensional slice and mesh plot of fuzzy total profit $\tilde{z}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3), \tilde{x}_1, \tilde{x}_2$ *and* $\tilde{x}_3$.

Table-3: Optimal Values for the Proposed Fuzzy Linear Programming Model

$\tilde{C}_j$			(4,6,7,8)	(5,8,9,10)	(5,6,8,9)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	**Min ratio**
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	
(0,0,0,0)	$\tilde{y}_4$	288	6	8	3	1	0	0	36→
(0,0,0,0)	$\tilde{y}_5$	312	12	8	6	0	1	0	39
(0,0,0,0)	$\tilde{y}_6$	124	2	$\boxed{4}$	1	0	0	1	31
$\tilde{z}_j$		(0,0,0,0)	(-8,-7,-6,-4)	(-10,-9,-8,-5)↑	(-9,-8,-6,-5)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	$\tilde{\Delta}_j$
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	**Min ratio**
(0,0,0,0)	$\tilde{y}_4$	40	2	0	1	1	0	-2	40
(0,0,0,0)	$\tilde{y}_5$	64	8	0	$\boxed{4}$	0	1	-2	16 →
(5,8,9,10)	$\tilde{y}_2$	31	$\frac{2}{4}$	1	$\frac{1}{4}$	0	0	$\frac{1}{4}$	124
$\tilde{z}_j$		(155,248,279,310)	$(-\frac{11}{2},-3,-\frac{3}{2},1)$	(0,0,0,0)	$(-\frac{31}{4},-6,-\frac{15}{4},-\frac{10}{4})$ ↑	(0,0,0,0)	(0,0,0,0)	$(\frac{5}{4},\frac{8}{4},\frac{9}{4},\frac{10}{4})$	$\tilde{\Delta}_j$
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	**Min ratio**
(0,0,0,0)	$\tilde{y}_4$	24	0	0	0	1	$-\frac{1}{4}$	$\boxed{\frac{3}{8}}$	64→
(5,6,8,9)	$\tilde{y}_3$	16	2	0	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	-
(5,8,9,10)	$\tilde{y}_2$	27	0	1	0	0	$-\frac{1}{16}$	$\frac{3}{8}$	72
$\tilde{z}_j$		(215,312,371,414)	(2,5,10,14)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	$(\frac{10}{16},\frac{15}{16},\frac{24}{16},\frac{31}{16}$	$(-\frac{21}{8},-1,\frac{3}{8},$ ↑	$\tilde{\Delta}_j$
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	**Min ratio**
(0,0,0,0)	$\tilde{y}_6$	0	0	0	0	$\frac{8}{3}$	$\frac{-2}{3}$	1	-
(5,6,8,9)	$\tilde{y}_3$	48	2	0	1	$\frac{4}{3}$	$\frac{-1}{12}$	0	-
(5,8,9,10)	$\tilde{y}_2$	3	0	1	0	$\frac{3}{16}$	$\frac{3}{16}$	0	-
$\tilde{z}_j$		(255,312,411,462) ≈ 360	(2,5,10,14) ≈ 7.77	(0,0,0,0)	(0,0,0,0)	$\left(\frac{365}{48},\frac{19}{2},\frac{593}{48},\frac{135}{8}\right.$ ≈ 11.58	$(\frac{3}{16},\frac{5}{6},\frac{19}{16},\frac{35}{24})$ ≈ 3.67	(0,0,0,0)	$\tilde{\Delta}_j \geq 0$

Comparison

In the fuzzy linear programming problem the optimal solutions are $\tilde{x}_1 = 0, \tilde{x}_2 = 3, \tilde{x}_3 = 48$ *and* $\tilde{z} = 360$ but for crisp linear programming problem the optimal solutions $x_1 = 0, x_2 = 27, x_3 = 16$ *and* $z = 312$. It can be observed that for fuzzy linear programming problem there will be more profit in comparison to crisp linear programming problem. So it is a cost effective technique and advisable to frame fuzzy linear programming problem for uncertain parameters.

CONCLUSIONS

In this paper Robust ranking technique has been implemented, for a linear programming problem with fuzzy parameters, which allows interactive decision in fuzzy decision space. The decision maker can intervene in all the steps of the decision process. The present approach will be very useful to be applied in a lot of real-world problems like production management, goal programming problems, integer programming problems and non linear programming problems for obtaining the optimal solution to draw managerial decision.

REFERENCES

1. Bellman, R.E and Zadeh, L.A. (1970). Decision making in a fuzzy environment. *Management Science*, 17: 141-164.
2. Zimmermann, H.J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1: 45-55.
3. Lai, Y.J. and Hwang, C.L. (1992). A new approach to some possibilistic linear programming problem. *Fuzzy Sets and Systems*, 49.

4. Lai, Y.J. and Hwang, C.L. (1992). *Mathematical Programming Methods and Applications*, Springer, Berlin.
5. Gani, A.N., Duraisamy, C. and Veeramani, C. (2009). A note on fuzzy linear programming problem using L-R fuzzy number. *International Journal of Algorithms, Computing and Mathematics*, 2 (3): 93-106.
6. Jimenez, M., Arenas, M., Bilbao, A. and Rodriguez, M.V. (2007). Linear programming with fuzzy parameters: An interactive method resolution. *European Journal of Operational Research*, 177 (3), 1599–1609.
7. Bazaraa, M.S., Jarvis, J.J. and Sherali, H.D. (1990). *Linear Programming and Network Flows*, John Wiley, Second Edition, New York.
8. Nasseri, S.H., Ardil, E., Yazdani, A., and Zaefarian, R. (2005). Simplex method for solving linear programming problems with fuzzy numbers. *World Academy of Science, Engineering and Technology*, 10: 284-288.
9. Rangarajan, R. and Solairaju, A. (2010). Computing improved fuzzy optimal Hungarian assignment problems with fuzzy costs under robust ranking techniques. *International Journal of Computer Applications*, 6(4): 6-13.
10. Pattnaik, M. (2012). *Models of Inventory Control*, Lambart Academic, Germany.
11. Swarup, K., Gupta, P.K. and Mohan, M. (2006). *Operations Research,* Sultan Chand and Sons, New Delhi.
12. Maleki, H.R. (2002). Ranking functions and their applications to fuzzy linear programming. Far. East Journal of Mathematical Science, 4: 283-301.
13. Maleki, H.R., Tata, M. and Mashinchi, M. (2000). Linear programming with fuzzy variables. Fuzzy Sets and Systems, 109: 21-31.
14. Rommelfanger, H. Hanuscheck, R., and Wolf, J. (1989). Linear programming with fuzzy objective. *Fuzzy Sets and Systems*, 29: 31-48.
15. Verdegay, J.L. (1984). A dual approach to solve the fuzzy linear programming problem. *Fuzzy Sets and Systems*, 14: 131-141.