

## Research Article

**Analysis of Head-loss Equations under EPANET and Hardy Cross Method**I.I. Nwajuaku<sup>1</sup>, Y.M. Wakawa<sup>2</sup>, O.J. Adibeli<sup>3</sup><sup>1,3</sup>Department of Civil Engineering, Nnamdi Azikiwe University, Awka Nigeria<sup>2</sup>Department of Civil Engineering Federal polytechnic Mubi, Nigeria**\*Corresponding Author:**

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**Abstract:** A comparative analysis of head-loss equations under EPANET computer program and Hardy Cross iterative method was carried out in this project. A single period analysis was performed on a three-loop network in EPANET's work space. Also, the network was solved with the Hardy Cross method, applying a single loop adjustment algorithm. The frictional head-losses in each of the methods were computed using the Darcy Weisbach and Hazen Williams head-loss equations. The convergence of the flow rate corrections in each of the cases was evaluated and compared. Results of the analysis showed that the flow rate corrections converged at the 4<sup>th</sup> and 14<sup>th</sup> iteration for the two head-loss formulae in EPANET and Hardy Cross method respectively. The results were subjected to t-test on 5% level of significance, and it showed that there is no significant difference in the use of either of the head-loss equations for computations in each of the methods. Thus, any one of the head-loss equations can be used for effective analysis of pipe networks.

**Keywords:** EPANET 2.0 software, Convergence, Head loss Equation, Hardy cross method, Pipe network.

**INTRODUCTION**

Increasing complexities associated with water distribution systems necessitated precise estimation of flows and pressures in various parts of the system. Solution of single pipe flow problems was no longer adequate. Quest for methods that analyze (solve for flows and pressures) for entire water distribution network gave birth to the topic "water distribution network analysis" or "pipe network analysis" [1]. For effective water distribution system, there is need for comparison among these design methods in order to get the method that will satisfactorily analyse a network system

Generally, there are four possible solution methods in pipeline analysis. These include loop equations, node-loop equations, node equations and pipe equations. The smallest set of equations is the loop equations that include one equation for each closed loop and pseudo-loop. The unknowns in the loop equations are the flow rate corrections which can be solved using either Hazen Williams or Darcy Weisbach head-loss equations. One of the earliest theories into finding solution to water flow and pressure in water distribution network includes the popular Hardy Cross method which is an iterative method for determining the flow in pipe network systems where the inputs and outputs are known, but the flow inside the network is unknown. A

number of subsequent works have appeared which further describe these methods or computer programs utilizing these methods [2, 1, 3]. The method was later made obsolete by computer solving algorithms employing Newton-Raphson method or other solution methods that removed the need to solve nonlinear systems of equations by hand. Many modelling programs are now available for commercial and educational use. Recently, several computer programs running on personal computers, such as EPANET, UNWB LOOP, WADISO, U of K KYPIPE, and WATER have been created and made available [4].

The EPANET software, developed by the USA Environmental Protection Agency (EPA) is commonly used because it is for general public and educational use and it is available free on-line. It is not only free but it requires relatively small computer space to operate. It has unlimited number of pipes that can be analyzed. EPANET applications in solving and/or optimizing water distribution network problems have been reported by [5-7]. The objective of this paper is to analyse a three-loop system using the Hardy Cross method and EPANET computer program while applying the head-loss equations in the two methods.

**MATERIALS AND METHODS**

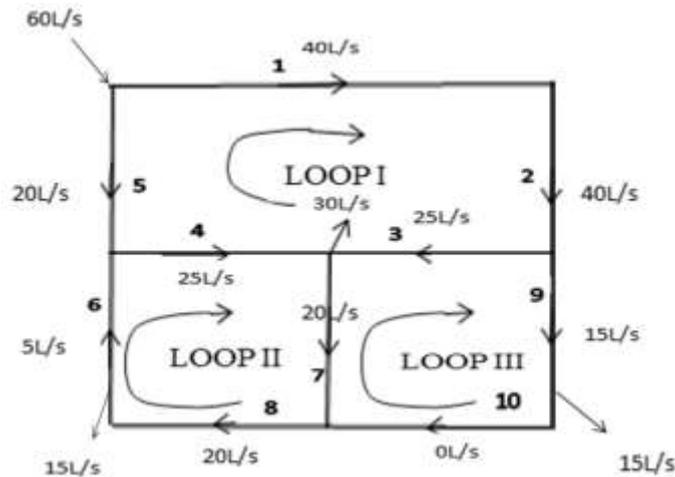
The materials used in this project were:

- EPANET-2.0 software [8].
- Microsoft Excel spreadsheet

**METHODS**

Two methods were used in this project for the pipe network analysis, namely: the EPANET

computer program and the Hardy Cross iterative method. Also, Darcy-Weisbach’s and Hazen William’s head loss equations are applied in the two methods. A simple distribution network shown in Figure 1 below was analysed. It consists of a three-looped pipe network with a source reservoir.



**Fig-1: The Sample Network with Initial the pipe flows.**

The following is the Hazen-Williams equation [9, 10].

Hazen William’s head loss:

$$H_L = \frac{10.675LQ^{1.852}}{C^{1.852}D^{4.8704}} \text{-----equation 1}$$

Where:

- C-factor = 120
- L = length of the pipe
- Q = flow rate in pipes
- D = pipe diameter

Darcy-Weisbach’s head loss

$$:H_L = \frac{8fLQ^2}{H^2gD^5} \text{-----equation 2}$$

Where:

- f = friction factor
- L=length of pipes
- Q=flow rate in pipes
- g=acceleration due to gravity (9.81m/s<sup>2</sup>)
- D=diameter of pipe

**Table 1 Pipe Properties**

Pipe	Length (m)	Diameter (mm)	Roughness Factor	
			D-W	H-W
1	250	300	0.025	120
2	100	200	0.025	120
3	125	200	0.025	120
4	125	200	0.025	120
5	100	200	0.025	120
6	100	150	0.025	120
7	100	150	0.025	120
8	125	250	0.025	120
9	100	150	0.025	120
10	125	250	0.025	120

**Table-2: Node Properties**

Node	Elevation (m)	Demand (L/s)
1	40	0
2	40	0
3	40	0
4	42	0
5	35	30
6	42	0
7	45	15
8	45	0
9	45	15

**Table-1: Pipe Properties**

Pipe	Length (m)	Diameter (mm)	Roughness Factor	
			D-W	H-W
1	250	300	0.025	120
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3	125	200	0.025	120
4	125	200	0.025	120
5	100	200	0.025	120
6	100	150	0.025	120
7	100	150	0.025	120
8	125	250	0.025	120
9	100	150	0.025	120
10	125	250	0.025	120

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Node	Elevation (m)	Demand (L/s)
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2	40	0
3	40	0
4	42	0
5	35	30
6	42	0
7	45	15
8	45	0
9	45	15

**EPANET 2.0 workspace**

The first task is to create a new project in EPANET and make sure that certain ID Label of the default options are selected as in fig. 2.

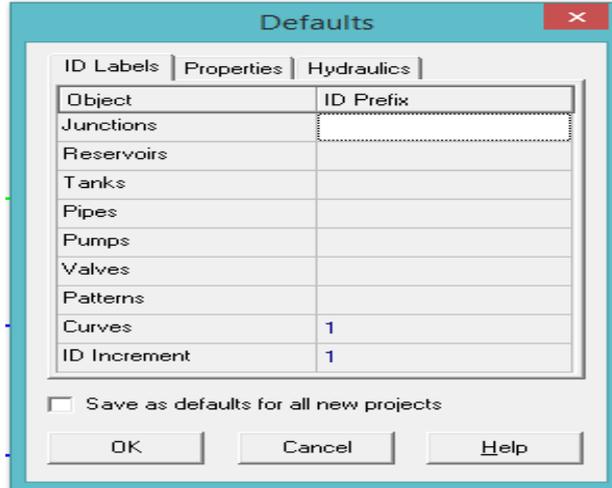


Fig-2: ID Label 1

The Hydraulics page of the dialog is also selected (as shown in Figure 3) and the choice of Flow Units set to LPS (litres per second).

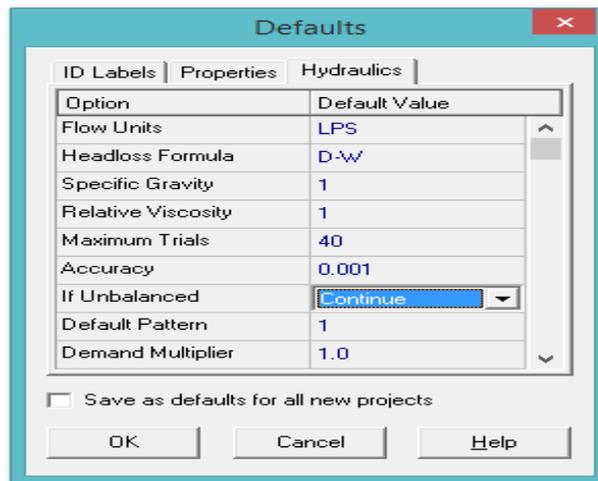


Fig-1: Hydraulic Page

The Notation page on this form is selected and the settings shown in Fig. 4 below are marked.

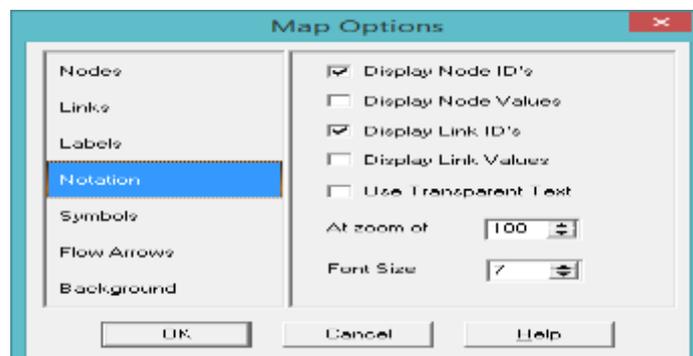
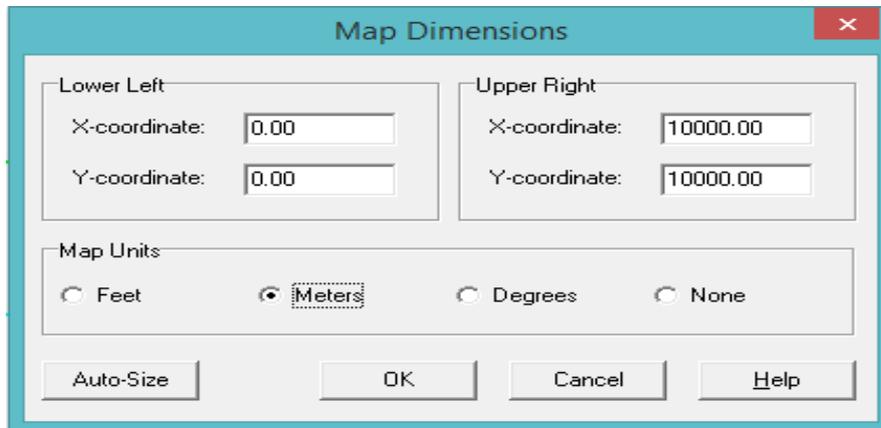


Fig-4: Map options dialog

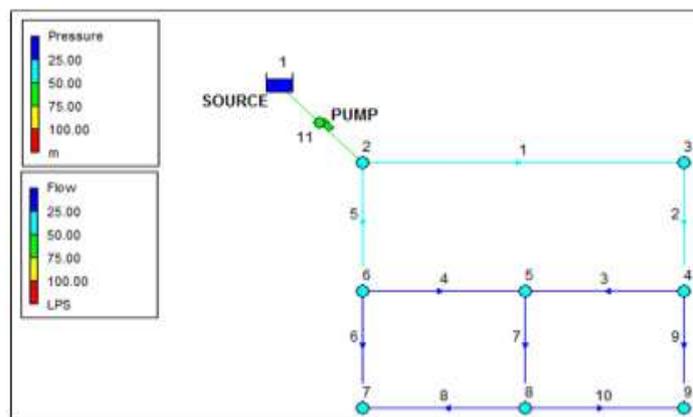
Map Dimensions dialog (as shown in Fig. 5 below), was selected from the main menu bar.



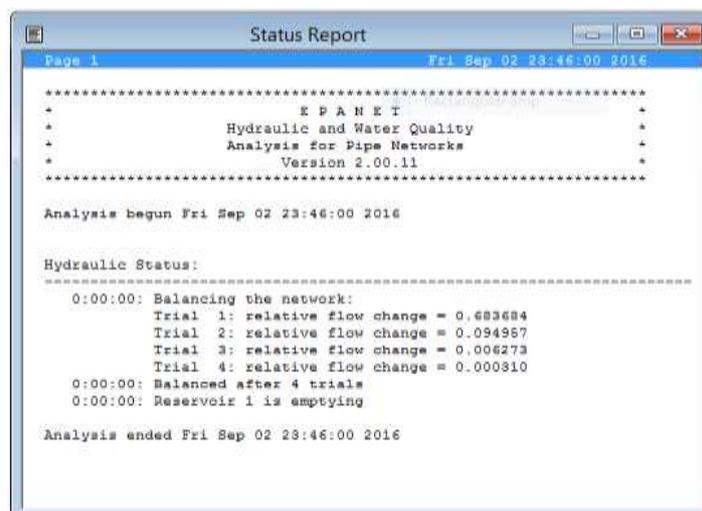
**Fig-5: Map dimensions dialog**

With the above setup completed, the network was drawn by making use of the Map Toolbar to add

pipes by connecting node 2 to node 10 and lastly the pump was added as shown in Fig. 6.



**Fig. 6: Network of the pipes**



**Fig-7: Status report**

Node ID	Elevation in	Base Demand LPS	Demand LPS	Head in	Pressure in
Junc 2	40	0	0.00	80.00	40.00
Junc 3	40	0	0.00	79.88	39.88
Junc 4	42	0	0.00	79.53	37.53
Junc 5	35	30	30.00	79.36	44.36
Junc 6	42	0	0.00	79.57	37.57
Junc 7	45	15	15.00	79.26	34.26
Junc 8	45	0	0.00	79.26	34.26
Junc 9	45	15	15.00	79.26	34.26
Reser 1	40	#N/A	-60.00	40.00	0.00

Fig-8: Network table nodes

Link ID	Length in	Diameter mm	Flow LPS	Velocity m/s	Unit Headloss m/km	Friction Factor
Pipe 1	250	300	28.39	0.40	0.49	0.018
Pipe 2	100	200	28.39	0.90	3.51	0.017
Pipe 3	125	200	17.02	0.54	1.37	0.018
Pipe 4	125	200	-19.28	0.61	1.73	0.018
Pipe 5	100	200	-31.61	1.01	4.29	0.017
Pipe 6	100	150	12.33	0.70	3.10	0.019
Pipe 7	100	150	6.30	0.36	0.92	0.021
Pipe 8	125	250	2.67	0.05	0.02	0.029
Pipe 9	100	150	11.37	0.64	2.67	0.019
Pipe 10	125	250	-3.63	0.07	0.03	0.027
Pump 11	#N/A	#N/A	60.00	0.00	-40.00	0.000

Fig-9: Network Table Links

**Hardy Cross iterative method**

The following is the loop equation for a closed loop [9,11]:

Loop Equation =  $\sum_{i=1}^N K_i |Q_i|^n \text{Sign}(Q)$  –equation (1)

Where:

$K_i = K_u \frac{L}{D^{4.87} C_{HW}^n}$  (for Hazen-Williams)

$Q_i$  = Flow through pipe “i” (m<sup>3</sup>/s)

N = Number of loops

n = Value based on which head-loss equation used (i.e. 1.852 for Hazen-Williams)

Equation 1 is essentially the sum of the head-loss in a predefined direction around a closed loop. Once the loop equation has been determined, the change in flow for that iteration must be calculated.

The equation for the change in flow for a closed loop [9, 10].

$\Delta Q_{LP} = - \frac{\sum_{i=1}^N K_i |Q_i|^n \text{Sign}(Q)}{\sum_{i=1}^N n |h_{L,i} / Q_i|}$  ----- equation. (2)

Where:

$\sum_{i \in \text{loop}} K_i |Q_i|^n \text{Sign}(Q)$  = Loop equation (sum of head-loss around a loop)

N = Number of loops

n = Value based on which head-loss equation used (i.e. 1.852 for Hazen-Williams),

**Test of Significant (t - test)**

Test of significance is a statistical tool used to give decisions to an argument. The procedure involves selecting a % of significance which is used to test the argument. The final decision of any argument is tied to the % of significance used. For example, if a 5% level of significance is chosen in designing a test of

significance, then there are about 5 chances in 100 that you will reject the hypothesis when it should be accepted, that is, we are about 95% confident that we have made the right decision. The t-statistics was adopted for this work because of the number of data points involved. Using statistical t-test for comparison of significant difference with a null ( $H_0$ ) and alternative ( $H_1$ ) hypotheses is as stated:

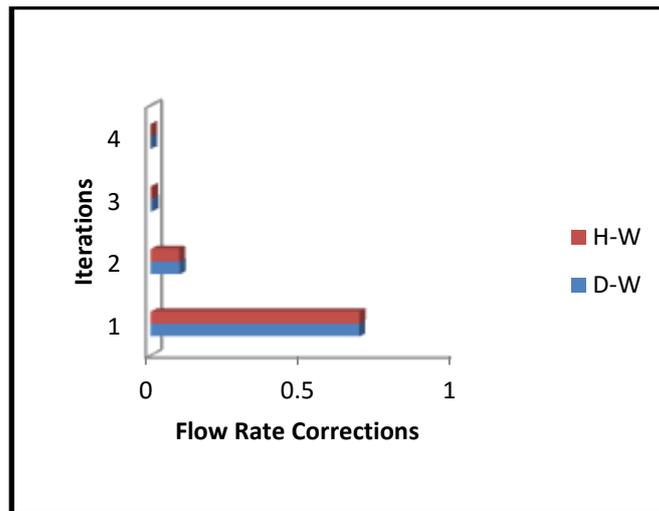
$H_0 = 0$ ; there is no significant difference between the two head-loss equations; and

$H_1 \neq 0$ ; there is a significant difference between the two head-loss equations.

**RESULTS AND DISCUSSION**

The results from the analysis of the two methods while applying Darcy Weisbach’s (D-W) and Hazen Williams’s (H-W) head-loss equations were presented and discussed below.

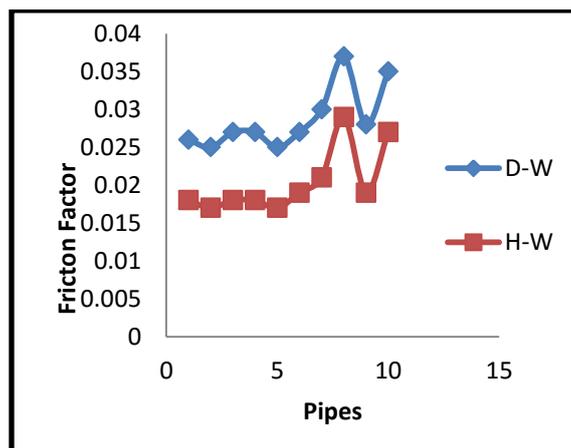
**Epanet output result**



**Fig-10: EPANET’s Output for Flow Rate**

Fig. 10 shows that the convergence of the flow rate for the pipe network occurred at the 4<sup>th</sup> iteration under the application of both the Darcy Weisbach and Hazen Williams head-loss equations. Closer

observation reveals that rate of convergence was faster when using the later equation than when using the former.



**Fig-11: EPANET’s output for friction factor**

Fig.11 above, shows that the friction factors which resulted in the application of the Darcy Weisbach head-

loss equation were higher than those obtained from the use of Hazen Williams’ formula

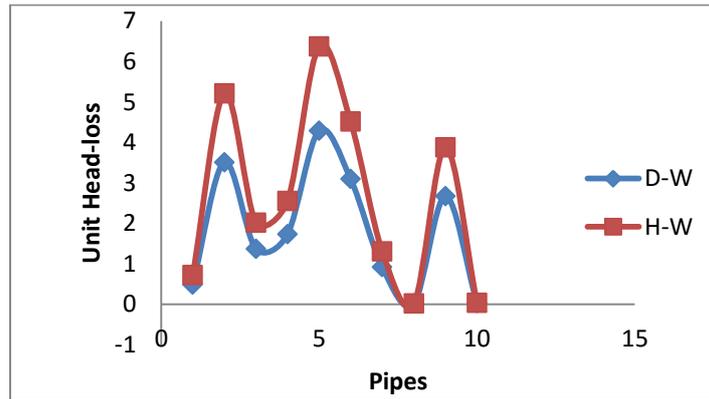


Fig-12: EPANET's Output for Unit Head loss

The variation of unit head-losses in all the pipes in the network is shown in Fig. 12 above. As can be seen from the above graph, the maximum head-loss occurred in pipe 5 under the application of both the

Darcy Weisbach and Hazen Williams head-loss formulae. The unit head-losses gotten from the use of the D-W formula are higher than those gotten from the H-W in all the pipes.

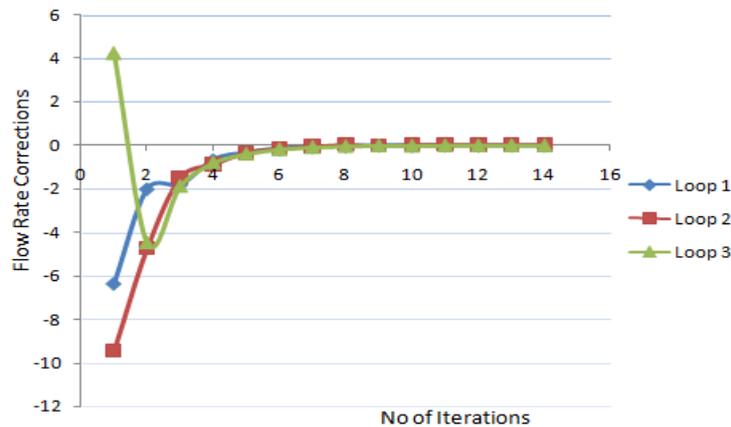


Fig-13: Plot of flow rate correction and no of iterations using D-W head-loss equation under Hardy Cross method.

Fig.13 shows the plot of flow rate correction against the number of iteration using Darcy- Weisbach head-loss equation. As supported by the findings of [11], since the initial solutions were by assumption, the values of the flow rate correction were high at the

beginning of the iteration process. They converged at the fourteenth (14<sup>th</sup>) iteration as shown in the plot. At that point, the values of flow rate obtained can be used for network analysis and design.

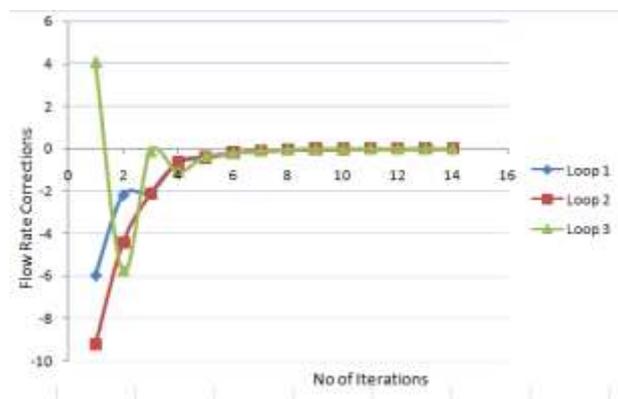


Fig-14: Plot of flow rate correction and number of iterations using Hazen Williams' head-loss equation

Also, as can be seen from Fig. 14 above, the plot of the flow rate corrections against the number of iterations, while using the Hazen Williams' head-loss equation, shows that the flow rate correction converged at the fourteenth (14<sup>th</sup>) iteration. The convergence in Figures 12 and 13 is almost the same because they are using a similar solution algorithm; i.e. the Hardy Cross single loop adjustment algorithm.

**Statistical Result**

**Statistical Comparison of the Results from Epanet**

A test of significance (t-test) was conducted between the Darcy Weisbach and Hazen Williams flow rate correction factors gotten above from EPANET. The standard error,  $S_d$  and the t-value are obtained from simulations as shown in Table 3 below.

**Table 3: T-Test Analysis between Darcy-Weisbach (D-W) and Hazen Williams (H-W) Flow Rate Corrections In Epanet**

Iteration	D-W	H-W	$d_i$	F <sup>2</sup>
1	0.683684	0.684965	-0.001281	3.96E-06
2	0.094957	0.093337	0.00162	8.29E-07
3	0.006273	0.004069	0.002204	2.23E-06
4	0.00031	0.000014	0.000296	1.71E-07
			0.002839	7.2E-06

From the Table,  $d_i = \text{Col.3} - \text{Col.4}$ ;

$F = d_i - \text{mean of } d_i$

$N = \text{No of samples} = 4$

Mean of  $d$  ( $d_{\text{mean}}$ ) =  $\frac{\sum d_i}{N} = 7.0975 \times 10^{-4}$

$S_d^2 = \frac{\sum (d_i - d_{\text{mean}})^2}{N-1} = 2.4 \times 10^{-6}$

$S_d = 1.549 \times 10^{-3}$

$t = \frac{d_{\text{mean}}}{S_d/\sqrt{N}} = \frac{2839}{3098} = 0.9164$

The number of degree of freedom ( $d_f$ ),  $v = N - 1 = 3$ .

The critical value of  $t$  or  $t_{\% \text{ sig}, d_f}$ , for 5% significance is  $t_{95, 3} = 3.182$ .

Given that  $t < t_{95, 3}$ , the null hypothesis is accepted. Thus, there is no significant difference in the use of the two head-loss equations for solving a water distribution network under EPANET.

**Statistical Comparison of the Results from the Hardy Cross Method**

Here, a test of significance was conducted between the flow rate correction factors ( $\Delta Q$ ) gotten at loop III of each iteration using both Darcy Weisbach and Hazen Williams head-loss equations.

The standard error,  $s_d$  and t-value are gotten from Table 4 below

**Table-4: T-Tests Analysis Between D-W and H-W Flow Rate Corrections in Hardy Cross**

Iteration	D-W	H-W	$d_i$	F <sup>2</sup>
1	4.248	4.077	0.171	0.027348
2	-4.484	-5.765	1.281	1.626572
3	-1.877	-0.131	-1.746	3.068203
4	-0.802	-1.091	0.289	0.080299
5	-0.392	-0.369	-0.023	0.00082
6	-0.173	-0.216	0.043	0.001397
7	-0.081	-0.105	0.024	0.000338
8	-0.038	-0.054	0.016	0.000108
9	-0.005	-0.028	0.023	0.000302
10	-0.018	-0.014	-0.004	9.27E-05
11	-0.004	-0.007	0.003	6.91E-06
12	-0.006	-0.004	-0.002	5.82E-05
13	0.0002	-0.002	0.0022	1.18E-05
14	0.001	-0.0006	0.0016	1.62E-05

From the above table,  $d_i = \text{Col.3} - \text{Col.4}$ ;

$F = d_i - \text{mean of } d_i$

$N = \text{No of samples} = 14$

Mean of  $d$  ( $d_{\text{mean}}$ ) =  $\frac{\sum d_i}{N} = 5.62857 \times 10^{-3}$

$S_d^2 = \frac{\sum (d_i - d_{\text{mean}})^2}{N-1} = 0.369659$

$S_d = 0.607996$

$t = \frac{d_{\text{mean}}}{S_d/\sqrt{N}} = \frac{2839}{3098} = 0.034639$

The number of degree of freedom ( $d_f$ ),  $v = N - 1 = 13$ .  
By reason of the alternative hypothesis,  $H_1$  a two tail test adopted. The critical value of  $t$  or  $t_{\% \text{ sigf, } d_f}$ , for 5% significance is  $t_{95, 13} = 2.160$

Given that  $t < t_{95, 13}$ , the null hypothesis is accepted. By implication, there is no significant difference in the use of the two head-loss equations for solving a water distribution network using Hardy Cross.

### CONCLUSION

There are many frictional head-loss equations which have been developed to determine frictional head-losses in pipelines. In line with the t-test result, no significant difference was observed between the outputs obtained from each of the head-loss equations. Hence, the two head-loss equations, Darcy Weisbach and Hazen Williams' equations can conveniently be used both in EPANET and Hardy Cross for the analysis of pipe network systems because they converge at the same iteration. Although rate of convergence was faster when using the H-W equation than when using the D-W.

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