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Original Research Article

A study of First geometric-Arithmetic Index with Chemical Applications

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Abstract: The first geometric–arithmetic index GA_1 is found to be useful in compute the numerical value of certain graphs and chemical graphs, which is defined as $GA_1 = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$. We denotes d_uthe degree of vertexuinG.In this paper,we present the general formula for the first geometric-Arithmetic Index for certain graphs and k_r-gluing graphs.
Keywords: First geometric-Arithmetic Index,k_r-gluing graph, complete graph, Web graph, wheel graph, k-bridge graph.

INTRODUCTION

Mathematical calculations are absolutely necessary to explore important concepts in chemistry. Mathematical chemistry is a branch of theoretical chemistry for discussion and prediction of the molecular structure using mathematical methods without necessarily referring to quantum mechanics. Chemical graph theory is an important tool for studying molecular structures. This theory had an important effect on the development of the chemical sciences one of these tools is topological indices [8, 10].

Topological indices are one of the main theoretical tools for studying molecular properties of chemical compounds. The definition of topological index is a numerical value associated with chemical constitution for correlation of chemical structure with various physical properties, chemical reactivity or biological activity [7, 16].

we need to recall a few concepts from chemical graph theory Let G be a molecular graph. Two vertices of G, connected by an edge, are said to be adjacent. If two vertices u and v are adjacent, we shall write $u \sim v$. The number of vertices of G, adjacent to a given vertex v, is the degree of this vertex, and will be denoted by $d_v(G)$, The concept of degree in graph theory is closely related (but not identical) to the concept of valence in chemistry [2,11-15].

First geometric-Arithmetic Index has been proposed in 2009 by D. VukiČeviĆ and B. Furtula. they analyzed some of its basic mathematical properties and proved that GA_1 to be better than well-known RandiĆ index. GA_1 is defined as :-

$$GA_1 = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

We denotes d_u the dgree of vertex u in graph G, The chemical applicability of the GA_1 index was examined and documented in detail in the paper [6], and the reviews [1,4].

Nowadays many new chemical compounds have been constructed and we need to study the chemical properties of these compounds. This process require a long time and expensive. So this paper is organized as follows. In Section 2, we specify the notation used and provide the necessary definitions. In Section 3, we will present the general formula for GA_1 Index to the Some Special Graphs,k-bridge graph, web graph, wheel graph. In Section 4 and 5 we obtain general formula for GA_1 Index to vertices gluing of graphs.

2. Basic Definition and Known Result

Suppose $G_1$and $G_2$are graphs each containing a complete sub–graph k_r ($r \geq 1$), Let G be a graph obtained from the union of $G_1$and G_2 by identifying the two subgraphs k_r in any arbitrary way as shown in figure 1. We call G a k_r-gluing graph of $G_1$and G_2, and denoted by $\varepsilon[G_1 \cup_r G_2]$ the family of all k_r -gluing graphof G_1 and G_2 . In particular where $r = 1$ ($resp.$ $r = 2$) we say G is vertex-gluing graph ($resp.$ an edge-gluing graph) of $G_1$and G_2.

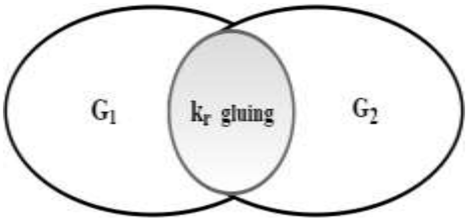

Fig-1: k_r-gluing graphof G_1 and G_2.

A graph is said to be complete if any two of its vertices are adjacent. The complete graph of order n is denoted by k_n (see Figure. 2).

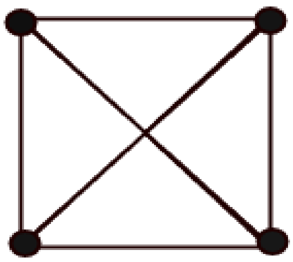

Fig-2: complete graph (k_4).

A web graph $W_{(n,m)}$ is the graph obtained from the Cartesian product of the cycle C_n and the path P_m (see Figure. 3).

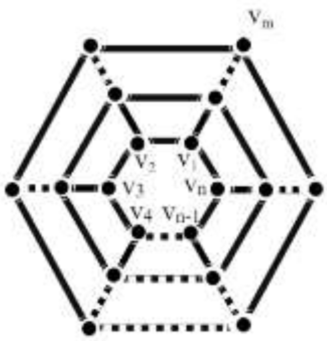

Fig-3: web graph $W_{(n,m)}$

A wheel W_n is a graph of order $n, n \geq 4$ obtained from the cycle C_{n-1} by adding a new vertex v adjacent to each vertex of the cycle i.e. $C_{n-1} + k_1$ (see Figure. 4).

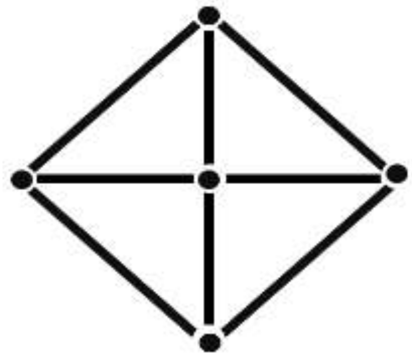

Fig- 4:W_5.

A graph consisting of s paths joining two vertices is called a k-bridge graph, which is denoted by $\theta(a_1, \ldots, a_k)$, where $a_1, \ldots, a_k$are the lengths of k paths. Clearly a k-bridge graphis a generalized polygon tree (see Figure.5).

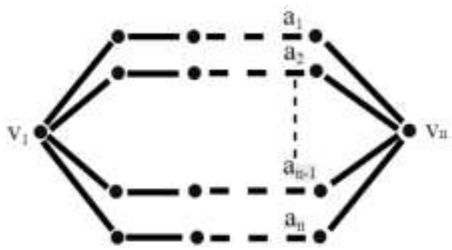

Fig-5: k-bridge graph.

We first give some examples of GA_1index for certain graphs. Let P_n, S_n, C_n and K_n be the path, star, cycle and complete graphs respectively with n vertices.

Example 2.1. Consider the complete graph k_n of order n. The first geometric–arithmetic index of this graph is computed as follow :-

$$GA_1(k_n) = \frac{n(n-1)}{2}.$$

Example 2.2. Suppose C_n is a cycle of length n labeled by $1,2, \ldots, n$. Then the first geometric–arithmetic index of this cycle is:-

$$GA_1(C_n) = N.$$

Example 2.3. If S_n is the star on n vertices, then

$$GA_1(S_n) = \frac{2\sqrt{(n-1)^3}}{n}.$$

Example 2.4. If P_n is the Path on n vertices, then

$$GA_1(p_n) = \begin{cases} 1. & if n = 2 \\ (n-3) + \frac{4\sqrt{2}}{3}. & if n \geq 3 \end{cases}$$

3. First Geometric-Arithmetic Index of Some Special Graphs

In this section, we present the general formulas for some special graphs.

Theorem 3.1. Let n, m be positive integers.then, the first geometric–arithmetic index of web graph $W_{(n,m)}$ is:-

$$GA_1\left(W_{(n,m)}\right) = \begin{cases} n(2m-1). & if m = 1,2 \ and \ n = 1,2, \ldots \\ n(2m-3) + 4n\frac{\sqrt{12}}{7}. & if m \geq 3 \ and \ n = 1,2, \ldots \end{cases}$$

Proof. We have two cases:

Case 1. If $m = 1$ or $m = 2$ then, We have cycles containing $n(2m-1)$ edges, and all edges in this case it have two vertices the same degree two or three.

Hence by the definition of first geometric–arithmetic index, we get:

theGA_1 for each one of all edges equal to one ($\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} = 1$), then

$$GA_1\big(W_{(n,m)}\big) = n(2m-1)\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v}.$$

$$GA_1\big(W_{(n,m)}\big) = n(2m-1)\cdot 1, \text{ then}$$

$$GA_1\big(W_{(n,m)}\big) = n(2m-1). \qquad if m = 1,2$$

Case 2. If $m \geq 3$ then, We have cycles containing $n(2m-3)$ edges, and all edges in this case it have two vertices the same degree three or four, and $(2n)$ of the edges link the vertices of cycles [all of them are incident on two vertices of degree three and four], then

$$GA_1\big(W_{(n,m)}\big) = n(2m-3) + 4n\frac{\sqrt{12}}{7} \qquad if m \geq 3.$$

Theorem 3.3. Let n, m be positive integers. Then, the first Geometric - Arithmetic Index of complete bipartite graph $k_{n,m}$ is

$$GA_1\big(k_{n,m}\big) = \frac{2(nm)\sqrt{nm}}{n+m}.$$

Proof. In complete bipartite graph $k_{n,m}$ there are nm edges, all of them are incident on two vertices of degree n and m.

Hence by the definition of first geometric–arithmetic index, we get:

$$GA_1\big(k_{n,m}\big) = \frac{2(nm)\sqrt{nm}}{n+m}.$$

Theorem 3.4. Let K be a positive integer, the first Geometric - Arithmetic Index of a $k-bridge$ graph denoted by $\theta(a_1, a_2, \dots, a_k)$ is:-

$$GA_1\big(\theta(a_1, a_2, \dots, a_k)\big) = (m-2k) + 4k\frac{\sqrt{2k}}{k+2}.$$

Proof. We have $(m-2k)$ edges, and all of them have two vertices the same degree two, and $(2k)$ of the edges are incident on two vertices of degree two and k, then

$$GA_1\big(\theta(a_1, a_2, \dots, a_k)\big) = (m-2k) + 4k\frac{\sqrt{2k}}{k+2}.$$

4. First Geometric-Arithmetic Index of Certain Vertex Gluing Graphs

In this section, we present the general formulas for first Geometric-Arithmetic Index to the vertices Gluing graphs.

Let G be the graph obtained from the vertex-gluing of two cycles C_n and C_m and denoted by $\varepsilon[C_n \cup_1 C_m]$, (see Figure. 6).

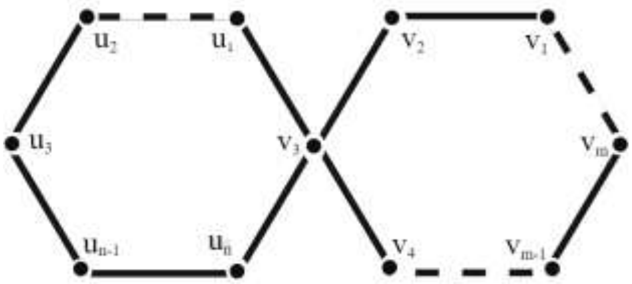

Fig-6: vertex-gluing of $\varepsilon[C_n \cup_1 C_m]$.

Theorem4.1. Letn, mbepositive integers. Then, the first Geometric - Arithmetic Index of the vertex-gluing of two cycles of $\varepsilon[C_n \cup_1 C_m]$is:-

$$GA_1\left(\varepsilon\left[C_n \bigcup_1 C_m\right]\right) = (n+m-4) + 8\frac{\sqrt{8}}{6}.$$

Proof. We have $(n+m-4)$ the numbers of edges, all of them are incident on a vertices the same degree two, and (4) of them are incident on two vertices of degree two and four, then

$$GA_1\left(\varepsilon\left[C_n \bigcup_1 C_m\right]\right) = (n+m-4) + 8\frac{\sqrt{8}}{6}.$$

Let G be the graph obtained from the edge-gluing of two cycles C_n and C_m and denoted by $\varepsilon[C_n \cup_2 C_m]$, (see Figure. 7).

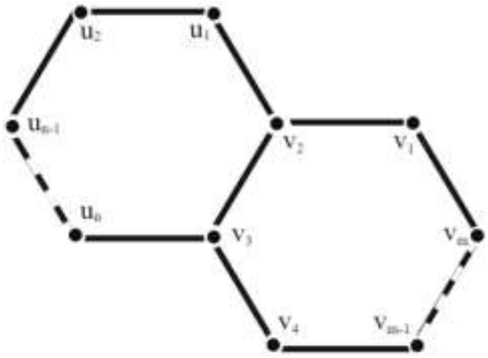

Fig-7: edge-gluing of $\varepsilon[C_n \cup_2 C_m]$

Theorem 4.2. Let n, m be positive integers. Then, the first Geometric - Arithmetic Index of the edge-gluing of two cycles $\varepsilon[C_n \cup_2 C_m]$ is:-

$$GA_1\left(\varepsilon\left[C_n \bigcup_2 C_m\right]\right) = (n+m-5) + 8\frac{\sqrt{6}}{5}.$$

Proof. We have $(n+m-5)$ the numbers of edges are incident on a vertices the same degree two, one of them incident on a vertices the same degree three, and (4) of them incident on a vertex of degree two and a vertex of degree three, then

$$GA_1\left(\varepsilon\left[C_n \bigcup_2 C_m\right]\right) = (n+m-5) + 8\frac{\sqrt{6}}{5}.$$

Let G be the graph obtained from the vertex-gluing of the two k -Bridge graphs θ_1 and θ_2 denoted by $\varepsilon[\theta_1 \cup_1 \theta_2]$, (see Figure. 8,9).

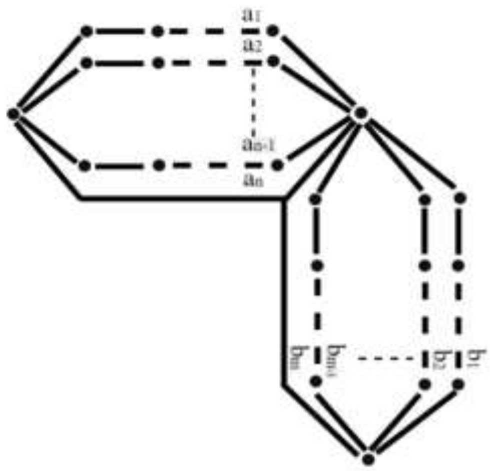

Fig-8: vertex -gluing by head of $\varepsilon[\theta_1 \cup_1 \theta_2]$.

Theorem 4.3. Let n, m be positive integers. Then, the first Geometric - Arithmetic Index of the vertex-gluing by head of $\varepsilon[\theta_1 \cup_1 \theta_2]$ is:-

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_1 \theta_2\right]\right) = \left(m - 2(k_1+k_2)\right) + 2k_1\frac{\sqrt{2k_1}}{k_1+2} + 2k_2\frac{\sqrt{2k_2}}{k_2+2} + 2(k_1+k_2)\frac{\sqrt{2(k_1+k_2)}}{k_1+k_2+2}.$$

Proof. We have $\left(m-2(k_1+k_2)\right)$ the numbers of edges, all of them have at least one vertex of degree two, (k_1) of them incident on a vertex of degree two and a vertex of degree k_1, (k_2) of them incident on a vertex of degree two and a vertex of degree k_2, and

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_1 \theta_2\right]\right) = \left(m-2(k_1+k_2)\right) + 2k_1\frac{\sqrt{2k_1}}{k_1+2} + 2k_2\frac{\sqrt{2k_2}}{k_2+2} + 2(k_1+k_2)\frac{\sqrt{2(k_1+k_2)}}{k_1+k_2+2}.$$

Theorem4.4. Let n, m be positive integers. Then, the first Geometric - Arithmetic Index of the vertex-gluing by side of $\varepsilon[\theta_1 \cup_1 \theta_2]$ is:-

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_1 \theta_2\right]\right) = \left(m-2(k_1+k_2)\right) + 4(k_1-1)\frac{\sqrt{2k_1}}{k_1+2} + 4(k_2-1)\frac{\sqrt{2k_2}}{k_2+2} + 4\frac{\sqrt{4k_1}}{k_1+4} + 4\frac{\sqrt{4k_2}}{k_2+4}. \ if\ a_n, b_1 = 2$$

$(k_1 + k_2)$ of them incident on a vertex of degree two and a vertex of degree $(k_2 + k_2)$.
Hence by the definition of first geometric–arithmetic index, we get:

Proof. We have $\left(m-2(k_1+k_2)\right)$ the numbers of edges, all of them have at least one vertex of degree two, $2(k_1-1)$ of them incident on a vertex of degree two and a vertex of degree k_1, $2(k_2-1)$ of them incident on a vertex of degree two and a vertex of degree k_2, (2) of them incident on a vertex of degree four and a vertex of degree k_1, and finally (2) of them incident on a vertex of degree four and a vertex of degree k_2.

Hence by the definition of first geometric–arithmetic index, we get:

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_1 \theta_2\right]\right) = \left(m-2(k_1+k_2)\right) + 4(k_1-1)\frac{\sqrt{2k_1}}{k_1+2} + 4(k_2-1)\frac{\sqrt{2k_2}}{k_2+2} + 4\frac{\sqrt{4k_1}}{k_1+4} + 4\frac{\sqrt{4k_2}}{k_2+4}. \ if\ a_n, b_1 = 2$$

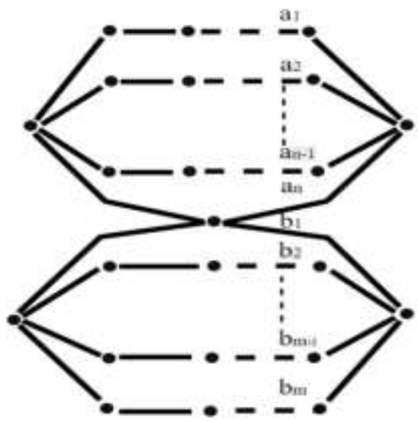

Fig-9: vertex-gluing by side of $\varepsilon[\theta_1 \cup_1 \theta_2]$.

Let G be the graph obtained from the edge-gluing of the two k-Bridge graphs θ_1 and θ_2 denoted by $\varepsilon[\theta_1 \cup_2 \theta_2]$, (see Fig.10,11,12,13).

Theorem4.5. Let n, m be positive integers. Then, the first Geometric - Arithmetic Index of the edge-gluing by head of $\varepsilon[\theta_1 \cup_2 \theta_2]$ is:-

1. If $a_n\ and\ b_m = 2,\ then$

$$GA_1\left(\varepsilon\left[\theta_1\bigcup_2\theta_2\right]\right) = (m-2(k_1+k_2)-1) + 2(k_1-1)\frac{\sqrt{2k_1}}{k_1+2} + 2(k_2-1)\frac{\sqrt{2k_2}}{k_2+2} + 2(k_1+k_2-2)\frac{\sqrt{2(k_1+k_2-1)}}{k_1+k_2+1} + 2\left(\frac{\sqrt{3(k_1+k_2-1)}}{k_1+k_2+2}+\frac{\sqrt{3k_1}}{3+k_1}+\frac{\sqrt{3k_2}}{3+k_2}\right).$$

2. If a_n and $b_m \geq 3$, then

$$GA_1\left(\varepsilon\left[\theta_1\bigcup_2\theta_2\right]\right) = (m-2(k_1+k_2)+1) + 2k_1\frac{\sqrt{2k_1}}{k_1+2} + 2k_2\frac{\sqrt{2k_2}}{k_2+2} + 2\frac{\sqrt{3(k_1+k_2-1)}}{k_1+k_2+2} + 2(k_1+k_2-2)\frac{\sqrt{2(k_1+k_2-1)}}{k_1+k_2+1} + 4\frac{\sqrt{6}}{5}.$$

3. If $a_n = 2$ and $b_m \geq 3$, then

$$GA_1\left(\varepsilon\left[\theta_1\bigcup_2\theta_2\right]\right) = \left(m-2(k_1+k_2)\right) + 2(k_1-1)\frac{\sqrt{2k_1}}{k_1+2} + 2\frac{\sqrt{3k_1}}{k_1+3} + 2k_2\frac{\sqrt{2k_2}}{k_2+2} + 2\frac{\sqrt{6}}{5} + 2(\mathrm{k}_1+\mathrm{k}_2-2)\frac{\sqrt{2(\mathrm{k}_1+\mathrm{k}_2-1)}}{\mathrm{k}_1+\mathrm{k}_2+1} + 2\frac{\sqrt{3(\mathrm{k}_1+\mathrm{k}_2-1)}}{\mathrm{k}_1+\mathrm{k}_2+2}.$$

Proof. We have three cases:

Case 1. We have a_i, $i = 1,2,3,\dots,n$, $a_n = 2$ and b_i, $i = 1,2,3,\dots,m$, $b_m = 2$ the numbers of edges. Therefore, we have $(m-2(k_1+k_2)-1)$ of them incident on a vertex of degree two, (k_1-1) of them incident on a vertex of degree two and a vertex of degree k_1, (k_2-1) of them incident on a vertex of degree two and a vertex of degree k_2, $(k_1+k_2-2)$of them incident on a vertex of degree two and a vertex of degree (k_1+k_2-1), one of them incident on a vertex of degree three and a vertex of degree (k_1+k_2-1), one of them incident on a vertex of degree three and a vertex of degree k_1, and finally one of them incident on a vertex of degree three and a vertex of degree k_2.
now by the definition of first geometric–arithmetic index, we get:

$$GA_1\left(\varepsilon\left[\theta_1\bigcup_2\theta_2\right]\right) = (m-2(k_1+k_2)-1) + 2(k_1-1)\frac{\sqrt{2k_1}}{k_1+2} + 2(k_2-1)\frac{\sqrt{2k_2}}{k_2+2} + 2(k_1+k_2-2)\frac{\sqrt{2(k_1+k_2-1)}}{k_1+k_2+1} + 2\left(\frac{\sqrt{3(k_1+k_2-1)}}{k_1+k_2+2}+\frac{\sqrt{3k_1}}{3+k_1}+\frac{\sqrt{3k_2}}{3+k_2}\right);\ \text{if } a_n, b_m = 2$$

Case 2. We have a_i, $i = 1,2,3,\dots,n$, $a_n \geq 3$ and b_i, $i = 1,2,3,\dots,m$, $b_m \geq 3$ the numbers of edges. Therefore, we have $(m-2(k_1+k_2)+1)$ of them incident on a vertex of degree two, k_1 of them incident on a vertex of degree two and a vertex of degree k_1, $k_2$of them incident on a vertex of degree two and a vertex of degree k_2, $(k_1+k_2-2)$of them incident on a vertex of degree two and a vertex of degree (k_1+k_2-1), one of them incident on a vertex of degree three and a vertex of degree (k_1+k_2-1), and finally two of them incident on a vertex of degree two and a vertex of degree three. Hence by the definition of first geometric–arithmetic index, we get:

$$GA_1\left(\varepsilon\left[\theta_1\bigcup_2\theta_2\right]\right) = (m-2(k_1+k_2)+1) + 2k_1\frac{\sqrt{2k_1}}{k_1+2} + 2k_2\frac{\sqrt{2k_2}}{k_2+2} + 2\frac{\sqrt{3(k_1+k_2-1)}}{k_1+k_2+2} + 2(k_1+k_2-2)\frac{\sqrt{2(k_1+k_2-1)}}{k_1+k_2+1} + 4\frac{\sqrt{6}}{5};\ \text{If } a_n \text{ and } b_m \geq 3$$

Case 3. We have a_i, $i = 1,2,3,\dots,n$, $a_n = 2$ and b_i, $i = 1,2,3,\dots,m$, $b_m \geq 3$ the numbers of edges. Therefore, we have $\left(m-2(k_1+k_2)\right)$ each one of them has two vertices that have the same degree two, $(k_1 - 1)$ of them incident on a vertex of degree two and a vertex of degree k_1, one of them incident on a vertex of degree three and a vertex of degree k_1, k_2 of them incident on a vertex of degree two and a vertex of degree k_2, one of them incident on a vertex of degree two and a vertex of degree three, $(k_1 + k_2 - 2)$ of them incident on a vertex of degree two and a vertex of degree $(k_1 + k_2 - 1)$, and finally one of them incident on a vertex of degree three and a vertex of degree $(k_1 + k_2 - 1)$.

Hence by the definition of first geometric–arithmetic index, we get:

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_2 \theta_2\right]\right) = \left(m - 2(k_1 + k_2)\right) + 2(k_1 - 1)\frac{\sqrt{2k_1}}{k_1 + 2} + 2\frac{\sqrt{3k_1}}{k_1 + 3} + 2k_2\frac{\sqrt{2k_2}}{k_2 + 2} + 2\frac{\sqrt{6}}{5} + 2(k_1 + k_2 - 2)\frac{\sqrt{2(k_1 + k_2 - 1)}}{k_1 + k_2 + 1} + 2\frac{\sqrt{3(k_1 + k_2 - 1)}}{k_1 + k_2 + 2} \text{ ; if } a_n = 2 \text{ and } b_m \geq 3$$

Theorem 4.6. Let n, m be positive integers. Then, the first Geometric - Arithmetic Index of the edge-gluing by side of $\varepsilon[\theta_1 \cup_2 \theta_2]$ is:-

If a_n and $b_m = 2$, then

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_2 \theta_2\right]\right) = m - 2(k_1 + k_2) + 4(k_1 - 1)\frac{\sqrt{2k_1}}{k_1 + 2} + 4(k_2 - 1)\frac{\sqrt{2k_2}}{k_2 + 2} + 4\frac{\sqrt{3k_1}}{k_1 + 3} + 4\frac{\sqrt{3k_2}}{k_2 + 3}.$$

Proof. In n, m-Bridge graph the edge-gluing by side of $\varepsilon[\theta_1 \cup_2 \theta_2]$ there are a_i, $i = 1,2,3,\ldots,n$, $a_n = 2$ and b_i, $i = 1,2,3,\ldots,m$, $b_m = 2$ edges. $m - 2(k_1 + k_2)$ the edges are also incident on a vertex of degreetwo, $2(k_1 - 1)$ of them incident on a vertex of degree two and a vertex of degree k_1, $2(k_2 - 1)$ of them incident on a vertex of degree two and a vertex of degree k_2, two of them incident on a vertex of degree three and a vertex of degree k_1, and finally two of them incident on a vertex of degree three and a vertex of degree k_2. now by the definition of first geometric–arithmetic index, we get:

$$GA_1\left(\varepsilon\left[\theta_1 \bigcup_2 \theta_2\right]\right) = m - 2(k_1 + k_2) + 4(k_1 - 1)\frac{\sqrt{2k_1}}{k_1 + 2} + 4(k_2 - 1)\frac{\sqrt{2k_2}}{k_2 + 2} + 4\frac{\sqrt{3k_1}}{k_1 + 3} + 4\frac{\sqrt{3k_2}}{k_2 + 3}.$$

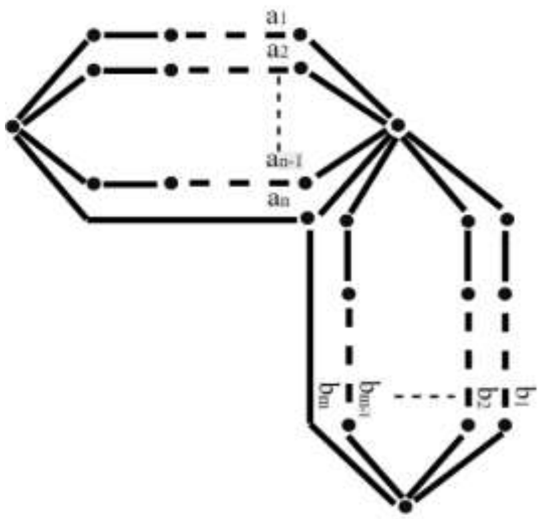

Fig-10: edge-gluing by head of $\varepsilon[\theta_1 \cup_2 \theta_2]$.

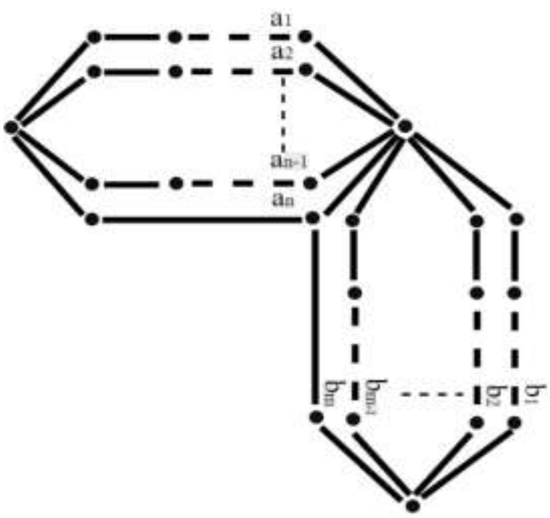

Fig-11:edge-gluing by head of $\varepsilon[\theta_1 \cup_2 \theta_2]$.

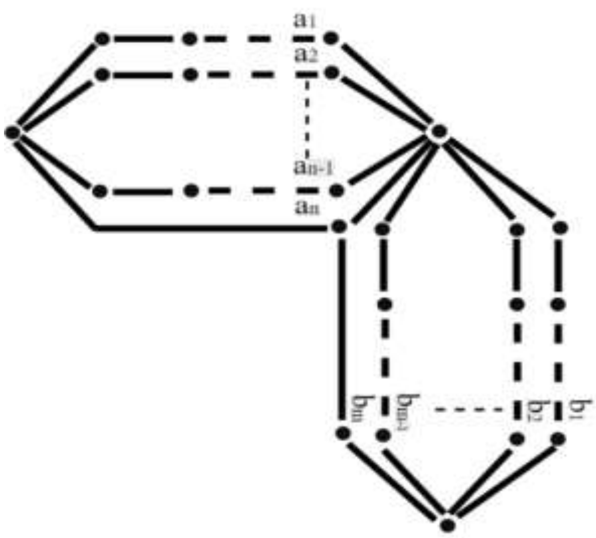

Fig-12:edge-gluing by head of $\varepsilon[\theta_1 \cup_2 \theta_2]$.

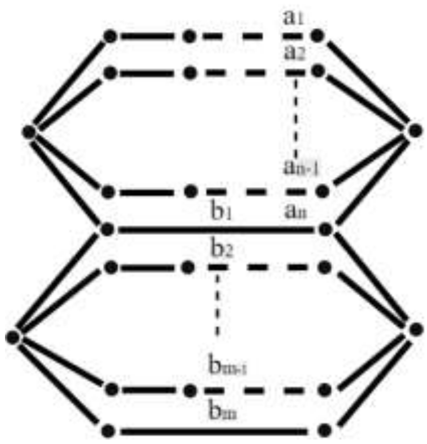

Fig-13: edge-gluing by side of $\varepsilon[\theta_1 \cup_2 \theta_2]$

5. First Geometric-Arithmetic Index of Some Special k_3-Gluing Graphs

In this section, we present the general formulas for first Geometric-Arithmetic Index for some special k_3 -Gluing graphs.

Let G be the graph obtained from the k_3-gluing of two graphs k_5 and W_n and denoted by $\varepsilon[k_5 \cup_3 W_n]$, (see Figure. 14).

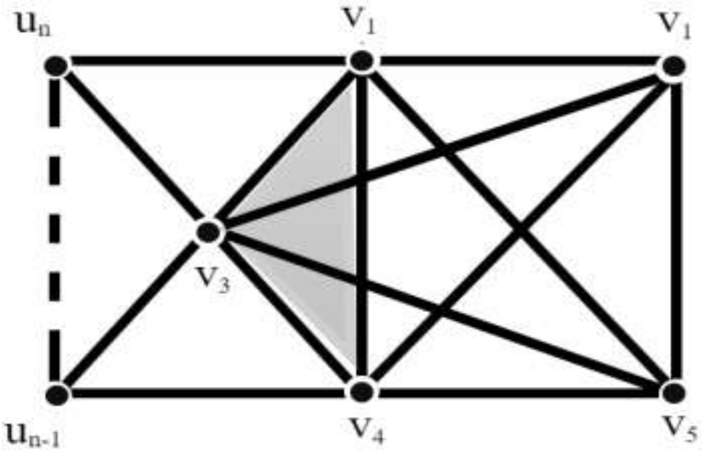

Fig-14: Figure 13:k_3-gluing of $\varepsilon[k_5 \cup_3 W_n]$.

Theorem5.1

Let n, m be positive integers. Then, the first Geometric-Arithmetic Index of k_3 -gluing of $\varepsilon[k_5 \cup_3 W_n]$ is:-

$$GA_1\left(\varepsilon\left[k_5 \bigcup_3 W_n\right]\right) = (n-2) + 4\frac{\sqrt{15}}{8} + 8\frac{\sqrt{20}}{9} + 4\left(\frac{\sqrt{4(n+1)}}{n+5} + \frac{\sqrt{5(n+1)}}{n+6}\right) + 2(n-3)\frac{\sqrt{3(n+1)}}{n+4}.$$

Proof. We have the edges of complete graph k_5 and wheel graph w_n, $(n-2)$ of them has two vertices that have the same degree, two of them incident on a vertex of degree three and a vertex of degree five, four of them incident on a vertex of degree four and a vertex of degree five, two of them incident on a vertex of degree four and a vertex of degree$(n+1)$,two of them incident on a vertex of degree five and a vertex of degree $(n+1)$, and finally $2(n-3)$ of them incident on a vertex of degree three and a vertex of degree $(n+1)$.

Hence by the definition of first geometric–arithmetic index, we get:

$$GA_1\left(\varepsilon\left[k_5\bigcup_3 W_n\right]\right) = (n-2) + 4\frac{\sqrt{15}}{8} + 8\frac{\sqrt{20}}{9} + 4\left(\frac{\sqrt{4(n+1)}}{n+5} + \frac{\sqrt{5(n+1)}}{n+6}\right) + 2(n-3)\frac{\sqrt{3(n+1)}}{n+4}.$$

Example 5.2

Let G be a graph of k_3-gluing of complete graph k_5 and wheel graph w_n , $n = 6$, then The first geometric–arithmetic index of the k_3-gluing of k_5 and w_6, is

$$GA_1\left(\varepsilon\left[k_5\bigcup_3 W_n\right]\right) = (6-2) + 4\frac{\sqrt{15}}{8} + 8\frac{\sqrt{20}}{9} + 4\left(\frac{\sqrt{4(6+1)}}{6+5} + \frac{\sqrt{5(6+1)}}{6+6}\right) + 2(6-3)\frac{\sqrt{3(6+1)}}{6+4}.$$

$$= 4 + 4\frac{\sqrt{15}}{8} + 8\frac{\sqrt{20}}{9} + 4\frac{\sqrt{28}}{11} + 4\frac{\sqrt{35}}{12} + 6\frac{\sqrt{21}}{10}.$$

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