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A Study on Bingham Plastic Characteristics of blood flow through multiple overlapped stenosed arteries

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Abstract: In this theoretical investigation, a mathematical model is developed to study the effect of multiple stenoses on flow characteristics of streaming blood through the atherosclerotic artery. The Bingham plastic fluid model of blood has been utilized in the study to represent the non-Newtonian character of blood. The geometry of the asymmetric shape of the stenosis assumed to be manifested in the arterial segment is given due consideration in the analysis. An extensive quantitative analysis is performed through numerical computations on flow resistance, wall shear stress and their variations are presented graphically for different stenotic and other rheological parameters. It is observed that the stenotic and physical parameters have considerable effect in the flow behaviour. Some important observations having medical interest on the flow of blood in the stenosed arteries are presented. The investigation bears the potential to explore a variety of information regarding some phenomenological aspects of the physiological problem. The output of the investigation may provide supplementary support to the physician in the treatment of the fatal disease.
Keywords: Stenosis, Bingham plastic model, flux, flow resistance, wall shear stress, shape parameter

INTRODUCTION

The cause and growth of many arterial diseases leading to serious circulatory disorder depend much on the flow characteristics and rheological properties of the streaming blood together with the mechanical behaviour of the blood vessel walls.

Arteries and veins are narrowed by abnormal and unusual deposition of cholesterols, fats, plaques etc in the inner lining of the arteries and veins resulting in the development of some cardiovascular diseases mainly atherosclerosis (medically termed stenosis.) Although the exact mechanism of the formation of stenosis in the arterial lumen is not understood from the standpoint of physiology/pathology, it is an established fact that the rheological and the fluid dynamic properties of blood and blood flow could play a significant role in the basic understanding, diagnosis and treatment of the arterial diseases. The tragedy of aging is that plaques build up and narrow arteries and this phenomenon make them stiffer and restricts (sometimes creates a blockade) blood flow in the arteries. The presence of stenosis in major blood vessels that supply blood to the brain, heart and other organs may lead to stroke, ischemia, heart attack, and many other cardiovascular diseases. Cardiac ischemia is caused due to the constriction responsible for insufficient flow of blood through the coronary arteries into the heart.

So, the investigations on flow characteristics of blood in multiple /overlapped stenosed arteries are of considerable importance when due attention is given to non symmetrical nature of the stenosis and non-Newtonian behaviour of blood. Many analytical as well as experimental studies confirmed that blood being a suspension of erythrocytes (red cells) in plasma, exhibits remarkable non-Newtonian behaviour when it flows through narrow arteries at low shear rates, particularly in diseased state.

A good number of analytical as well as experimental studies on blood flow through the arterial segments having a stenosis or a multiple stenosis were carried out by several investigators applying different fluid model for blood and various types of stenotic geometry. H-B fluid model and Casson fluid models are generally accepted in the theoretical study of blood flow through narrow arteries. The mathematical analysis presented by Sankar [2] discusses the pulsatile flow of blood through stenosed narrow artery treating blood as H-B fluid. Bali et al presented a theoretical analysis to study the response of externally applied magnetic field on the flow of blood through multiple stenosed artery assuming the Casson fluid model for blood. Yadav *et al.* [1] investigated the effects of the length of the stenosis and shape parameter on the resistance to flow through an artery with multiple stenosis at equal distances. Blood is characterized as Bingham plastic fluid model.

It is assumed that the wall of vessel is almost rigid and so the non-Newtonian Bingham plastic viscous incompressible fluid model will be appropriate to represent blood and blood flow in the human artery. On the basis of the experimental observations it is established that stenosis may develop in series and may also overlap. The geometry of the multiple non-symmetrical stenosis manifested in the arterial segment is given due weightage considering the appropriate mathematical expression. The proposed study will be devoted to present some new characteristics of blood flow in stenosed arteries which were not given due attention by previous investigators though they may have a dominant role in the diagnosis and treatment of this fatal disease.

MATERIALS & METHODS

For mathematical convenience an artery with overlapping stenosis symmetrical about the axis but non symmetrical with respect to radial coordinates is considered. The geometry of the stenosis assumed to be manifested in the arterial segment is given by the mathematical expression

$$\frac{R}{R_0} = \begin{cases} 1 - \varepsilon[L_0^{s-1}\{\gamma z - nd - (n-1)L_0\} - \{\gamma z - nd - (n-1)L_0\}^s], & n(d+L_0) - L_0 \le \gamma z \le n(d+L_0) \\ 1, & \text{otherwise} \end{cases} \quad --(i)$$

R, R_0 : Tube radius with and without stenosis

n : Number of stenosis in the artery

L_0 : Stenosis length

d : Location of the stenosis

s : Stenosis shape parameter and $s \geq 2$

r : A positive constant ≥ 1

$$\varepsilon = \frac{\delta}{R_0} \cdot \frac{s}{L_0^s (s-1)} - - - - - - - - (ii) \qquad \delta$$

:Maximum height of the stenosis at $z = \dfrac{nd + (n-1)L_0 + L_0 / s^{1/s-1}}{\gamma} - - - - (iii)$

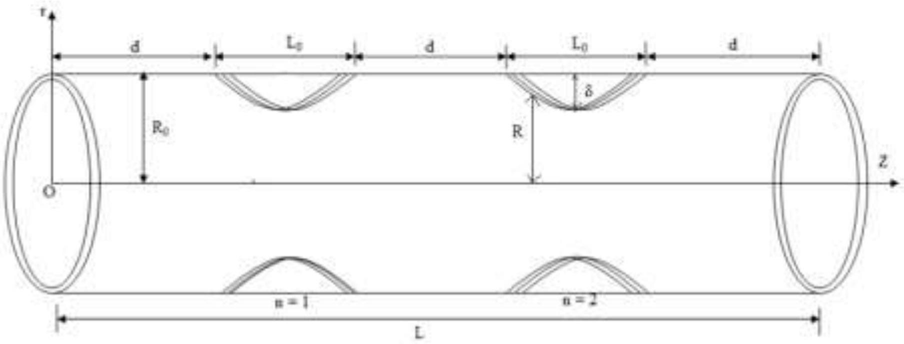

Fig-1: Geometry of the Artery with stenoses

The governing equation for Bingham plastic fluid model is

$$\dot{\beta} = f(\tau) = -\frac{du}{dr} = \begin{cases} \frac{\tau - \tau_0}{\mu}, & \tau > \tau_0 \\ 0, & \tau \le \tau_0 \end{cases} \quad ----(iv)$$

The mathematical expression for flux Q through the artery is given by

$$Q = \int_0^R 2\pi\, r\, u(r)\, du \quad \_\_\_\_\_\_\_\_(\quad v)$$

BOUNDARY CONDITIONS & MATHEMATICAL SOLUTION:

$u = 0$ at $r = R$ (no slip) $------(vi)$

τ is finite at $r = 0 ----------(vii)$

At the wall i.e., when $r = R$, the expressions for shear stress in terms of the pressure p are given by

$$\tau = -\frac{r}{2}\frac{dp}{dz} \;\&\; \tau_R = -\frac{R}{2}\frac{dp}{dz} \Rightarrow \frac{\tau}{\tau_R} = \frac{r}{R} ------(viii)$$

Integrating (v) with boundary conditions (vii) $Q = \pi\int_0^R r^2 (-\frac{du}{dr})\, dr -----(ix)$ (Using the principle of integration by parts)

With the help of (iv), $Q = \pi\int_0^R r^2 f(\tau)\, dr ----(x)$

Using (vi) $Q = \pi\frac{R^3}{\tau_R^3}\int_0^{\tau_R} \tau^2 f(\tau)\, d\tau = \pi\frac{R^3}{\tau_R^3}\int_0^{\tau_R} \tau^2 \frac{\tau - \tau_0}{\mu} d\tau$ $Q = \frac{\pi}{\mu}\frac{R^3}{\tau_R^3}[\frac{\tau_R^4}{4} - \tau_0\frac{\tau_R^3}{3}] -----(xi)$

The wall shear stress τ_R is given by $\tau_R = \frac{4Q\mu}{\pi R^3} + \frac{4}{3}\tau_0 -----(xii)$

The second part of equation $(viii)$ gives $\frac{dp}{dz} = -\frac{2}{R}\tau_R = -\frac{2}{R}\left[\frac{4Q\mu}{\pi R^3} + \frac{4}{3}\tau_0\right]$

$$\frac{dp}{dz} = -\frac{8Q\mu}{\pi R^4} - \frac{8\tau_0}{3R} ------(xiii)$$

If $p = p_1$ for $z = L$ and $p = p_0$ for $z = 0$, then integrating equation $(xiii)$ with respect to z

$$p_1 - p_0 = -\frac{8Q\mu}{\pi R_0^4}\int_0^L \left(\frac{R}{R_0}\right)^{-4} dz - \frac{8\tau_0}{3R_0}\int_0^L \left(\frac{R}{R_0}\right)^{-1} dz$$

The resistance to flow λ is given by

$$\lambda = \frac{p_1 - p_0}{Q} = -\frac{8\mu}{\pi R_0^4}\int_0^L \left(\frac{R}{R_0}\right)^{-4} dz - \frac{8\tau_0}{3QR_0}\int_0^L \left(\frac{R}{R_0}\right)^{-1} dz$$

$$= -f_1\int_0^L \left(\frac{R}{R_0}\right)^{-4} dz - f_2\int_0^L \left(\frac{R}{R_0}\right)^{-1} dz ------(xiv)$$

where $f_1 = \frac{8\mu}{\pi R_0^4}$ and $f_2 = \frac{8\tau_0}{3QR_0}$

$$= -f_1\left\{\int_0^{\frac{n(d+L_0)-L_0}{\gamma}}\left(\frac{R}{R_0}\right)^{-4}dz+\sum_{n=1}^{n=n_{\max}}\int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}}\left(\frac{R}{R_0}\right)^{-4}dz+\int_{\frac{n(d+L_0)}{\gamma}}^{L}\left(\frac{R}{R_0}\right)^{-4}dz\right\}$$

$$-f_2\left\{\int_0^{\frac{n(d+L_0)-L_0}{\gamma}}\left(\frac{R}{R_0}\right)^{-1}dz+\sum_{n=1}^{n=n_{\max}}\int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}}\left(\frac{R}{R_0}\right)^{-1}dz+\int_{\frac{n(d+L_0)}{\gamma}}^{L}\left(\frac{R}{R_0}\right)^{-1}dz\right\}----(xv)$$

Taking $n = 2$, we get

$$\lambda=-f_1\left\{\int_0^{\frac{2d+L_0}{\gamma}}dz+\int_{\frac{d}{\gamma}}^{\frac{d+L_0}{\gamma}}\left(\frac{R}{R_0}\right)^{-4}dz+\int_{\frac{2d+L_0}{\gamma}}^{2\left(\frac{d+L_0}{\gamma}\right)}\left(\frac{R}{R_0}\right)^{-4}dz+\int_{2\left(\frac{d+L_0}{\gamma}\right)}^{L}dz\right\}-$$

$$f_2\left\{\int_0^{\frac{2d+L_0}{\gamma}}dz+\int_{\frac{d}{\gamma}}^{\frac{d+L_0}{\gamma}}\left(\frac{R}{R_0}\right)^{-1}dz+\int_{\frac{2d+L_0}{\gamma}}^{2\left(\frac{d+L_0}{\gamma}\right)}\left(\frac{R}{R_0}\right)^{-1}dz+\int_{2\left(\frac{d+L_0}{\gamma}\right)}^{L}dz\right\}------(xvi)$$

$$\lambda=-f_1\left\{\frac{2d+L_0}{\gamma}-2\left(\frac{d+L_0}{\gamma}\right)+L\right\}-f_2\left\{\frac{2d+L_0}{\gamma}-2\left(\frac{d+L_0}{\gamma}\right)+L\right\}-(f_1I_1+f_2I_2)-(f_1J_1+f_2J_2)$$

$$\Rightarrow \lambda=-(f_1+f_2)(L-\frac{L_0}{\gamma})-(f_1I_1+f_2I_2)-(f_1J_1+f_2J_2----(xvii))$$

where $I_1=\int_{\frac{d}{\gamma}}^{\frac{d+L_0}{\gamma}}\left(\frac{R}{R_0}\right)^{-4}dz$ *and* $J_1=\int_{\frac{2d+L_0}{\gamma}}^{2\left(\frac{d+L_0}{\gamma}\right)}\left(\frac{R}{R_0}\right)^{-4}dz$

$I_2=\int_{\frac{d}{\gamma}}^{\frac{d+L_0}{\gamma}}\left(\frac{R}{R_0}\right)^{-1}dz$ *and* $J_2=\int_{\frac{2d+L_0}{\gamma}}^{2\left(\frac{d+L_0}{\gamma}\right)}\left(\frac{R}{R_0}\right)^{-1}dz$

In the absence of any stenosis $L_0=0, I_1=I_2=J_1=J_2=0$

$\lambda_N=-(f_1+f_2)L$

The resistance to flow is given by

$$\bar{\lambda}=\frac{\lambda}{\lambda_N}=1-\frac{L_0}{L\gamma}+\frac{f_1(I_1+J_1)+f_2(I_2+J_2)}{(f_1+f_2)L}-----(xviii)$$

From (xii), the wall shear stress $\tau_R=\dfrac{4Q\mu}{\pi R^3}+\dfrac{4}{3}\tau_0----(xix)$

In the normal condition $\tau_N=\dfrac{4Q\mu}{\pi R_0^3}----(xx)$

The wall stress ratio is given by $\bar{\tau}_R = \frac{\tau_R}{\tau_N} = \frac{1}{(R/R_0)^3} + \frac{\pi R_0^{\,3}\tau_0}{3Q\mu} - - - (xxi)$

At the mid-point of the second stenosis, $R = R_0 - \delta,\ \bar{\tau}_R = \frac{1}{\left(1-\frac{\delta}{R_0}\right)^3} + \frac{\pi R_0^{\,3}\tau_0}{3\mu Q} - - (xxii)$ For $n = 2$, the result (i) become $\frac{R}{R_0} = 1 - \varepsilon\left[L_0^{\,s-1}(\gamma z - 2d - L_0) - (\gamma z - 2d - L_0)^s\right] - - - (xxiii)$

where $\varepsilon = \frac{\delta}{R_0} . \frac{s^{s/(s-1)}}{L_0^{\,s}(s-1)} - - - - (xxiv)$

and δ is the maximum height of the stenosis at $z = \frac{2d + L_0 + L_0 / s^{1/s-1}}{\gamma} - - - - (xxv)$

NUMERICAL RESULTS & DISCUSSION:

To discuss the output of the analysis quantitatively, algorithms and computer codes are developed for the numerical evaluations of the analytical results given in equations $(xviii)$ & (xxi).

The purpose of this numerical computation is to bring out the effects of parameters on shear stress and flow resistance in the flow behaviour of blood through the diseased artery taking the Bingham plastic model for blood.

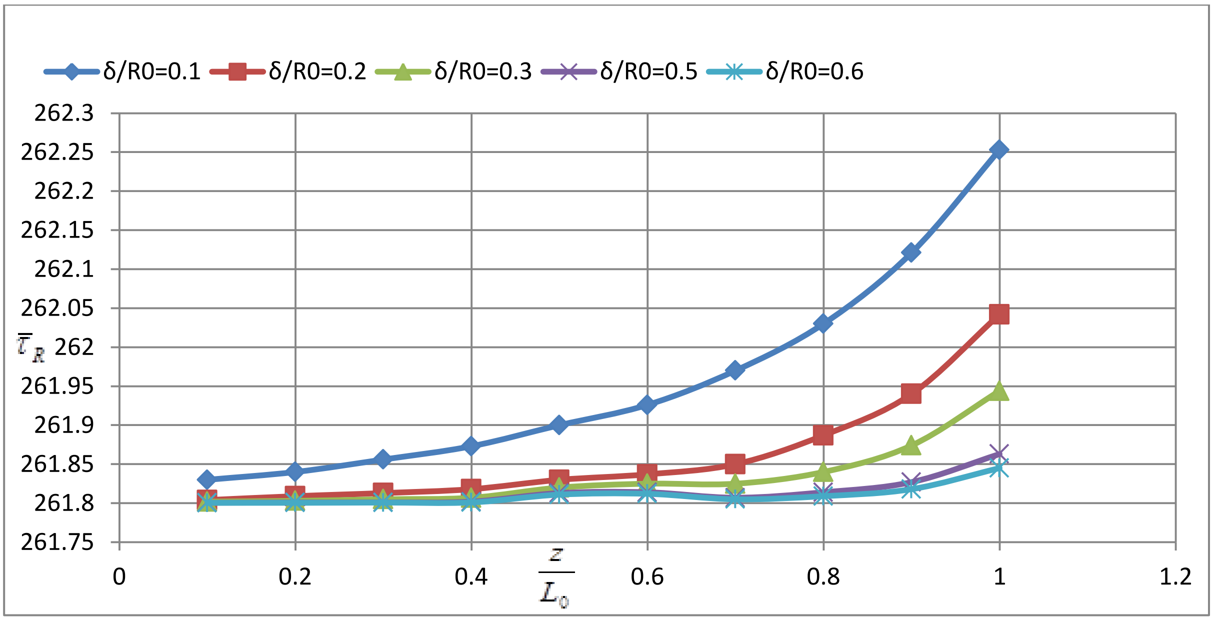

Fig-2: Shear stress $\bar{\tau}_R$ versus $\frac{z}{L_0}$ for different values of stenosis height $\left(\frac{\delta}{R_0}\right)$

The variation of shear stress with the axial distance in the stenosed portion of the artery is presented in Fig. 2 for stenosis height in the range (0.1-0.6). The shear stress increases with the axial distance. However the magnitude is noted to be steeply rising for higher value of stenosis height. It confirms that stenosis height has considerable influence in the generation of shear stress in the stenosis region. The result is in close agreement with the physical nature of the problem under investigation.

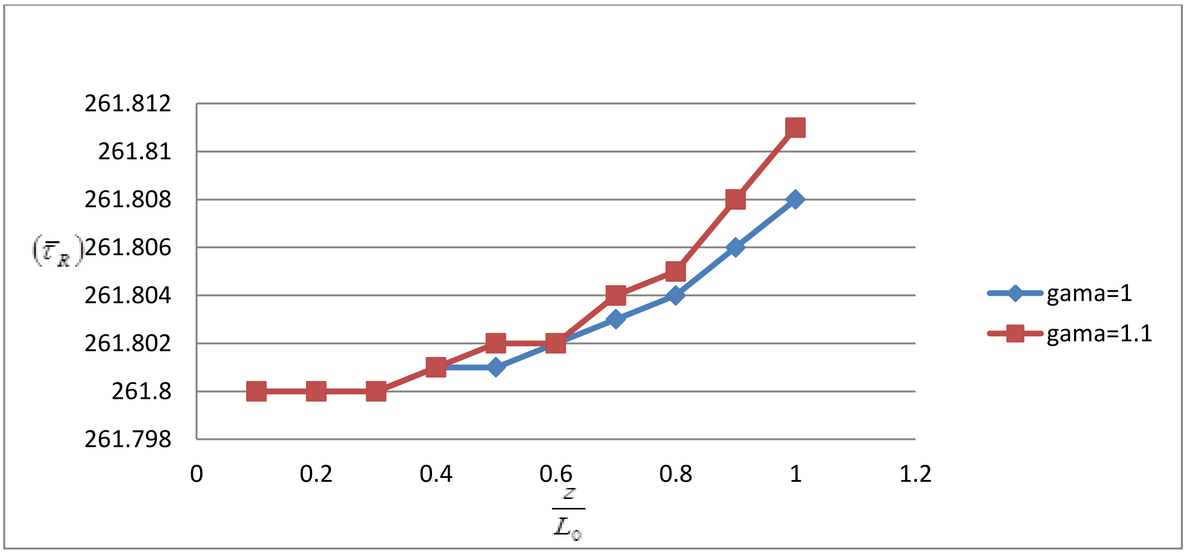

Fig-3: Shear stress $(\bar{\tau}_R)$ versus $\frac{z}{L_0}$ for different values of Gamma (γ)

The variation of shear stress with the axial distance in the stenosed portion of the artery is exhibited in Fig. 3 for $\gamma = 1$ & 1.1. Though the shear stress increases with the axial distance for both values of γ, but the parameter has some role for higher axial distance, though the contribution is not dominant. The results for both the parameters almost converge for lower values of the axial distance.

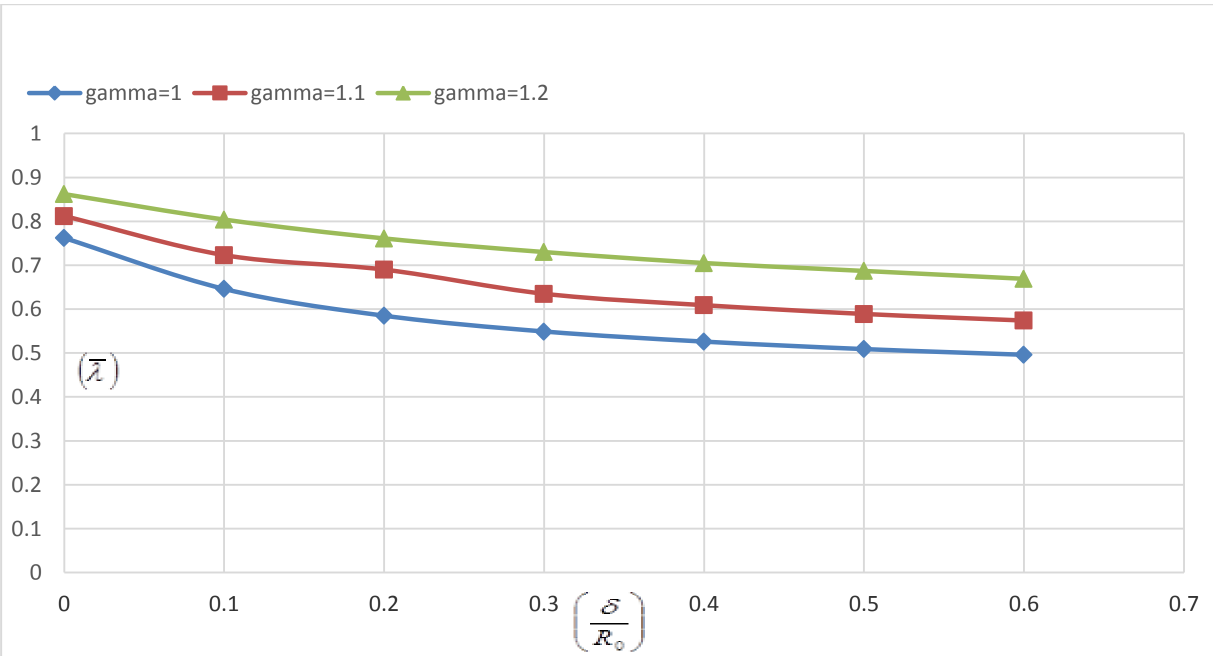

Fig-4: Flow resistance $(\bar{\lambda})$ versus stenosis height $\left(\frac{\delta}{R_0}\right)$ for different values of parameter Gamma (γ)

The distribution of the flow resistance $(\bar{\lambda})$ developed in the diseased artery with stenosis height $\left(\frac{\delta}{R_0}\right)$ is demonstrated in Fig. 4 for the parameter $\gamma = 1, 1.1, 1.2$. It shows similar trend for the values of γ under consideration.

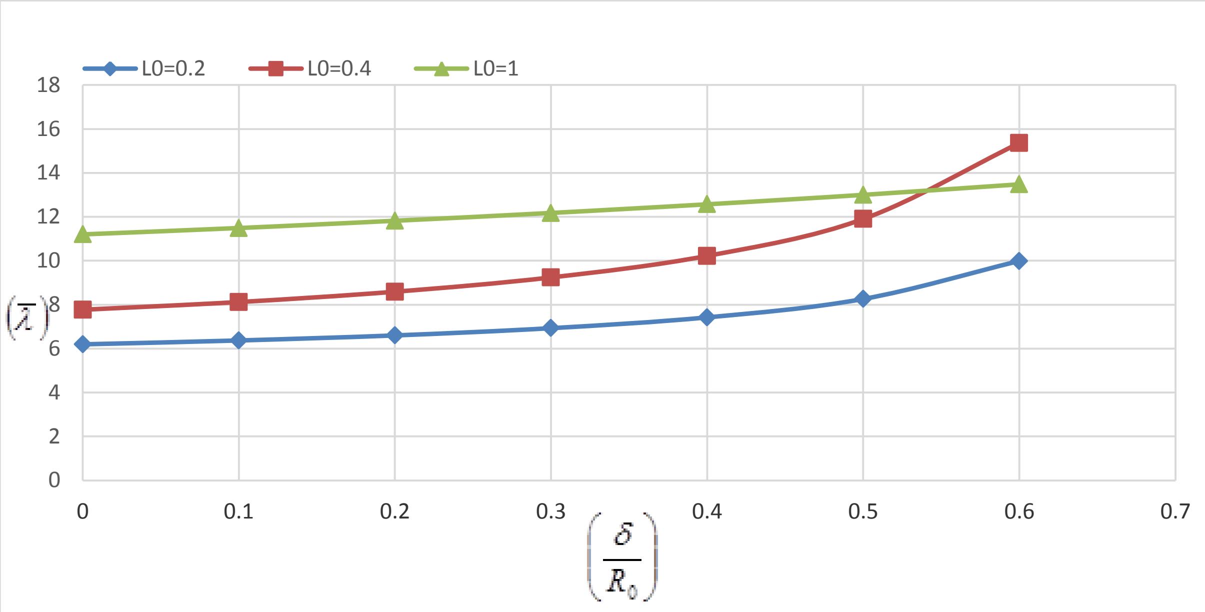

Fig-5: Flow resistance $(\bar{\lambda})$ versus stenosis height $\left(\frac{\delta}{R_0}\right)$ for different values of L_0

Fig. 5 exhibits the variation of flow resistance $(\bar{\lambda})$ with stenosis height $\left(\frac{\delta}{R_0}\right)$ taking the stenosis length (L_0) in the range 0.2 – 1. The flow resistance follows similar pattern in the range. In each case, the flow resistance gradually increases with the increase in the value of stenosis height which is quite significant. As the length of stenosis (L_0) increases from 0.2 – 1, the flow resistance gradually increases for the values of $\left(\frac{\delta}{R_0}\right)$ under consideration excepting at $L_0 = 0.4$ which shows a rapid increasing pattern in the range (0.5-0.6) for $\frac{\delta}{R_0}$. The rapid growth may be accepted as $L_0 = 0.4$ lies in the vicinity of the middle of the stenosis.

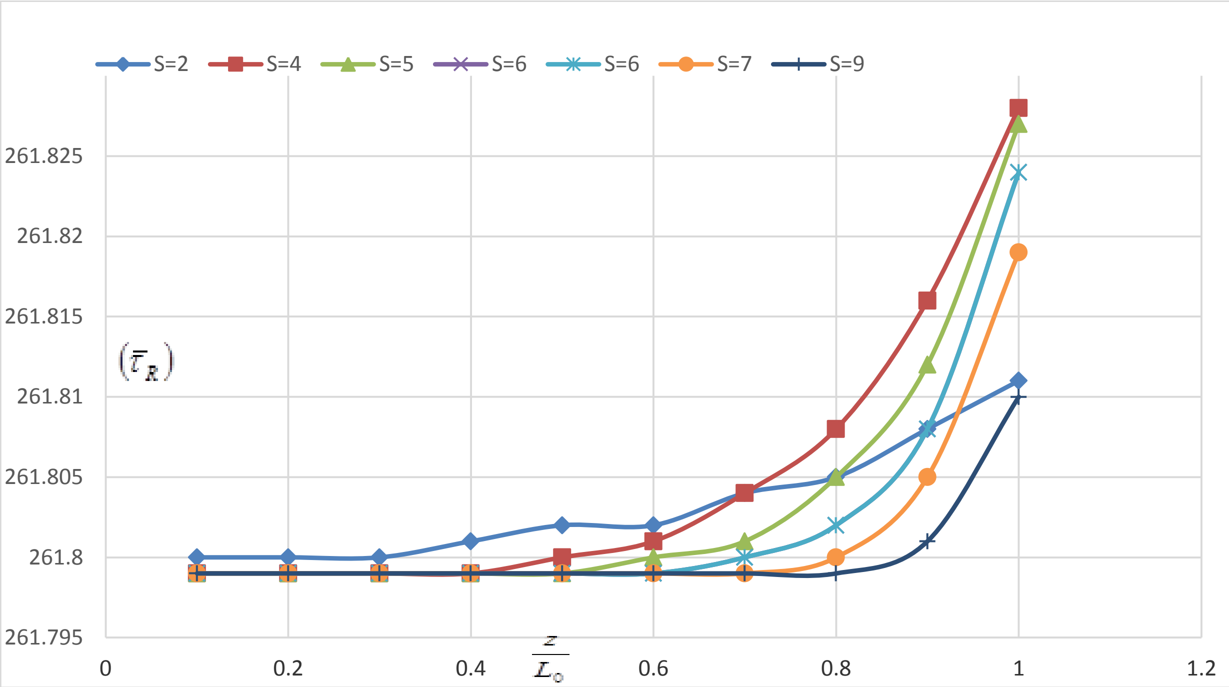

Fig-6: Shear stress $(\bar{\tau}_R)$ versus $\frac{z}{L_0}$ for different values of stenosis shape parameter (s)

Fig. 6 illustrates the variation of shear stress $(\bar{\tau}_R)$ for different values of $\frac{z}{L_0}$ with stenosis shape parameter s in the range (2-9), the restriction being $s \geq 2$. The shear stress shows increasing pattern for all values of $\frac{z}{L_0}$ but steeply increases for $\frac{z}{L_0}$ in the range (0.6-1). It is further observed that at any given position of the stenosis, the shear stress shows increasing trend for the increase of the value of the stenosis shape parameter(s) and the increase is more significant and also physically acceptable.

CONCLUSIONS

A number of interesting conclusions can be drawn from the Article:

(i) An analytical model may provide an enhanced understanding of the key parameters that control the system.

(ii) The consideration of the two layered solid-fluid structure provides confidence in the analysis.

(iii) The output of the theoretical investigation may provide supplementary support to the physician in the treatment of the fatal disease.

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