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Expression of Energy in Special Relativity and Newton kinetic Energy formula

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Abstract: This work discussed expression of the energy and millennium relativity of energy in Einstein generalized special relativity and pression of direct modification of Newton kinetic energy by using Taylor series he four vectors energy momentum of total energy formula is derived through energy relation and Lorontz transformation.
Keywords: Millennium Relativity (MR), Special Relativity (SR)

INTRODUCTION

Atoms and elementary particles are described by quantum mechanicalLaws However Einstein general relativity is a generally covariant theory of gravity[1]. Direct modification of Newton's kinetic energy formula is conducted in the fewest steps practical in the derivation of the millennium relativity form of the relativistic kinetic energy formula. The resultant formula is then analyzed for improved understanding of the relativistic principles involved[2,3]. Einstein theory of special relativity (SR) is one of the biggest achievements in modern. It changes radically the classical concept of space and time [4] such generalization was made in the Einstein generalized special relativity (EGSR) model [5]. In this model general relativity (GR) was used to find a useful expressions for time, length mass and energy to cure the fore noted defects [6, 7]. In this paper we discuss in details how to express the energy in Special Relativity and Newton kinetic Energy formula by different methods.

Direct modification of Newton kinetic energy

Newtonian kinetic energy formula is given by the form:

$$k = \frac{1}{2} mv^2 \ldots\ldots\ldots\ldots(1)$$

Newtonian kinetic energy And the millennium relativity gamma factor in the form

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{c}{v}\right)^2}} = \frac{c}{\sqrt{c^2-v^2}}\ldots \quad (2) \qquad \text{(MR) gamma factor}$$

Form equation (1) and equation (2) we derive when $\gamma = 1$

$$k = \frac{1}{1+\gamma} m(v\gamma)^2 \ldots\ldots(3) \quad \text{(Relativistic kinetic energy)}$$

Form equation (3) derive the kinetic energy we get

$$k = \frac{1}{1+\gamma} mv^2\gamma^2 \ldots\ldots\ldots(4)$$

Relativistic kinetic energy form the momentum

According the relation between energy (E) and mass (M) in special relativistic theory the energy is given by

$$E = mc^2 \ldots\ldots\ldots\ldots\ldots(5)$$

Using the relation between mass (m) and rest mass (m_0)

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Equation (6) means that the particle mass increase with the increased speed substitute equation (6) in equation (5) we get

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0c^2}{\sqrt{\frac{m^2c^4 - m^2v^2c^2}{m^2c^4}}} = m_0\gamma^2 \ldots\ldots\ldots (7)$$

The momentum component (p) is given by

$$p = mv \ldots\ldots\ldots\ldots\ldots (8)$$

Also the relativistic momentum

$$\mathrm{p} = \mathrm{mv}\gamma$$

Substitute equation (8) and equation (5) in equation (7) we get

$$E = \frac{m_0c^2}{\sqrt{\frac{E^2 - P^2C^2}{E^2}}} = \frac{m_0c^2E}{\sqrt{E^2 - P^2C^2}} \ldots\ldots\ldots\ldots\ldots (9)$$

$$\sqrt{E^2 - P^2C^2} = m_0c^2$$

$$E^2 = P^2C^2 + m^2c^4 \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Relativistic kinetic and total energy

Substitute the right side of equation (2) form gamma in equation (5) we derive

$$k = \frac{1}{1+\frac{c}{\sqrt{c^2-v^2}}} mv^2 \frac{c^2}{\sqrt{c^2-v^2}}$$

$$k = \frac{mvc}{\sqrt{\sqrt{c^2-v^2}}}\left(\frac{vc}{\sqrt{\sqrt{c^2-v^2}+c}}\right) \ldots\ldots\ldots\ldots (11)$$

$$k = \frac{mv^2c^2}{c\sqrt{c^2-v^2}+c^2-v^2} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The equation (12) MR total energy

Millennium relativity kinetic energy (MRKE)

Then add the internal energy term mc^2 to millennium relativistic kinetic energy equation in equation (12) we get millennium relativity kinetic energy (MRKE)

$$E = \frac{mc^2v^2}{c\sqrt{c^2-v^2+c^2-v^2}} + mc^2(MRTE) \ldots\ldots\ldots (13)$$

Relativistic momentum by newton 2^{nd} law

Form the newton 2^{nd} law can be written in the form

$$\mathrm{F} = \frac{\mathrm{dp}}{\mathrm{dt}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Where the new relativistic momentum of a body is $p = mv$

$$v = \frac{dx}{dt}$$

Suppose $$p = m\frac{dx}{d\tau} \ldots\ldots\ldots\ldots\ldots (15)$$

Where (τ) is the proper time in the object rest forme Also since $y^{-} = y$ and $z^{-} = z$ the transverse momentum $(p_y \; and \; p_z)$ will be invariant for alocnt z transformation along the x axis we can rewrite this momentum definition as follows

$$p = m\frac{dx}{d\tau} = m\frac{dx}{d\tau}.\frac{dt}{d\tau}$$

$$t = \gamma\tau \; \rightarrow \; \frac{dt}{d\tau} = \gamma$$

Form the time dilation

$$p = \gamma_v mv \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

$$\gamma_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} \ldots\ldots\ldots..(17)$$

V is the velocity of the object in reference frameNot the velocity of a reference frame relation of another .Form the equation (1) and equation (15) and using the Newtonian definitions of energy and momentum

Let $E = \frac{1}{2}mv^2 \ and \ p = mv$

We can write

$$E = \frac{p^2}{2m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Now consider the relativistic definitions

$$E = \gamma mc^2$$

$$p = \gamma mv$$

$$p^2 = \gamma^2 m^2 v^2 \ldots\ldots\ldots\ldots(19)$$

Multiplying both side by c^2 yieds

$$p^2c^2 = \gamma^2 m^2 v^2 c^2 = \gamma^2 m^2 c^4 \frac{v^2}{c^2}$$

$$p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right)$$

$$p^2c^2 = \gamma^2 m^2 c^4 - m^2c^4$$

But $\quad E = \gamma mC^2 \ldots\ldots\ldots\ldots\ldots\ldots(20)$

Then $\quad p^2c^2 = E^2 - m^2c^4$

Thus the equivalent relationship between energy and momentum in relation is

$$E^2 = p^2c^2 + m^2c^4 \ldots\ldots\ldots\ldots(21)$$

This equation is smeller of equation (10) and equation (13) MRTE

Correct at sufficiently low speeds. A definition for the relativistic momentum of particle moving with velocity And form the energy equation

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0c^2}{\sqrt{\frac{m^2c^4 - m^2v^2c^2}{m^2c^4}}}$$

$$E = \frac{m_0c^2}{\sqrt{\frac{E^2-P^2c^2}{E^2}}} = \frac{m_0c^2E}{\sqrt{E^2-P^2c^2}} \qquad (22)$$

$$\sqrt{E^2-P^2c^2} = m_0c^2, \qquad E^2 - P^2c^2 = {m_0}^2c^4$$

$$E^2 = P^2c^2 + {m_0}^2c^4 \qquad (23)$$

$$pd\tau = mdx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots(24)$$

Also we define the relativistic momentum of particle by:

$$p = \frac{m_0v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \ldots\ldots\ldots\ldots\ldots..(25)$$

The lab frame time $(T)$and proper time (τ) are related through

$$dt = \frac{d\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \ldots\ldots\ldots\ldots\ldots..(26)$$

Where $d\tau$ is the time elapsed on the particle dt is the time elapsed in the laboratory and (v) is speed of the particle in the laboratory so that in some particle inertial frame

$$r = [ct, x, y, z]$$

And

$$p = m\frac{dr}{d\tau} = m\frac{dt}{d\tau}.\frac{dr}{dt} = \frac{m_0}{\sqrt{\sqrt{1-\left(\frac{v}{c}\right)^2}}}(c, v_x, v_y, v_z)$$

The last three components of this four –vector are easy to interpret.
They say that the relativistic momentum in particular frame is defined as

$$\mathrm{p} = \frac{m_0 \mathrm{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The initial component

$$\mathrm{p} = \frac{mc}{\sqrt{1-\left(\frac{\mathrm{v}}{\mathrm{c}}\right)^2}}$$

Form the equation (20) this is proportional to relativistic energy

Total energy by four – vector in relativity

Form the relativity –space and time coordinates and energy, momentum of particle sometimes expressed by four vectors to fine total energy

Special cases of the total energy relation

$$\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2 + m_\circ{}^2\mathrm{c}^4$$

1- if the body in to mass less particle $m_{0=0}$ then $\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2$

2- if the body s speed v is much less c, then $E^2 = \frac{1}{2}m_\circ v^2 + m_\circ{}^2c^4$

3- if the body is at rest (v=0) in its center of momentum frame p=0 we have $m = m_\circ$ and $E = E_\circ$

The square of a vector $p^2 = p.p = |p|^2$

Let as

$$E = \gamma_v m_\circ c^2,\ p = \gamma_v m_\circ v \quad \text{and} \quad m = \gamma m_\circ$$

Then $$p^2 = p.p = \frac{m_\circ{}^2 v^2}{1-\frac{v^2}{c^2}} \ldots\ldots\ldots.. (28)$$

Solving v^2 and substituting in to the Lorentz vector we get

$$\gamma = \sqrt{1+\frac{p^2}{m_\circ{}^2\,c^2}}$$

Let $p = \left(\frac{E}{c}, p\right)$ in special relativity

$$\mathrm{E} = m_\circ{}^2\,c^2\sqrt{1+\frac{p^2}{m_\circ{}^2\,c^2}} \ldots\ldots\ldots\ldots. (29)$$

Using the Minkowski metric (Ƞ) the liner product is

$$< p, p >= |p|^2 = -(m_\circ c)^2$$

<, > the minkowski liner product

$$< p, p > = p^a \eta_{ab} \mathrm{p}^\mathrm{b} = \left(\frac{\mathrm{E}}{\mathrm{c}}\mathrm{p_x p_y p_z}\right)\begin{bmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\frac{E}{c}\\\mathrm{p_x}\\\mathrm{p_y}\\\mathrm{p_z}\end{bmatrix} = -\left(\frac{\mathrm{E}}{\mathrm{c}}\right)^2 + \mathrm{p}^2$$

$$\mathrm{p}^2 = -\left(\frac{\mathrm{E}}{\mathrm{c}}\right)^2 + \mathrm{p}^2$$

So

$$(m_\circ c)^2 = -\left(\frac{\mathrm{E}}{\mathrm{c}}\right)^2 + \mathrm{p}^2 \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Also the space – time 4 vectors and energy – momentum 4 vector is defined by

$$\vec{R} = \begin{bmatrix}\mathrm{ct}\\\mathrm{x}\\\mathrm{y}\\\mathrm{z}\end{bmatrix} = \begin{bmatrix}\mathrm{ct}\\\vec{\mathrm{r}}\end{bmatrix}$$

$$\vec{p} = \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix} = \begin{bmatrix} E \\ \vec{p}c \end{bmatrix}$$

For the scalar product space and time 4- vector

$$\overrightarrow{R_b} = \begin{bmatrix} ct \\ \vec{r}_b \end{bmatrix} \quad , \overrightarrow{R_a} = \begin{bmatrix} ct \\ \vec{r}_a \end{bmatrix}$$

$$R_a . R_b = ct_a ct_b - \vec{r}_a . \vec{r}_b \text{..................}(31)$$

And the scalar product of two energy momentum 4- vector by

$$\overrightarrow{p_a} = \begin{bmatrix} E_a \\ p_a c \end{bmatrix} \quad , \quad \overrightarrow{p_b} = \begin{bmatrix} E_b \\ p_b c \end{bmatrix}$$

$$\overrightarrow{p_a} . \overrightarrow{p_b} = E_a . E_b - \overrightarrow{p_a} . \overrightarrow{p_b} c^2 \dots \dots \dots \dots \dots (32)$$

Using the length of space – time 4- vector and length of energy – momentum

$$\vec{R} . \vec{R} = (CT)^2 - (x^2 + y^2 + z^2)$$

And

$$\sqrt{p.p} = \sqrt{E^2 - (PC)^2} = m_\circ c^2 \quad \dots \dots \dots \dots \dots (33)$$

Rewriting equation (23) for massive particles

Using the Taylor series

$$(1 + \varepsilon)^n = 1 + n\varepsilon + \frac{1}{2} n(n - 1)\varepsilon^2 + \cdots$$

$$E = m_\circ c^2 \left[1 + \frac{1}{2} \left(\frac{p}{m_\circ c} \right)^2 - \frac{1}{8} \left(\frac{p}{m_\circ c} \right)^4 + \dots \dots \dots . \right]$$

In the limit v<<c we have $\gamma(v) \approx 1$ so the momentum has classical form

$p \approx m_\circ v$ Then to first order in $\left(\frac{p}{m_\circ c} \right)^2$ is retain the term $\left(\frac{p}{m_\circ c} \right)^{2n}$ for n=1 and neglect all terms for $n \geq 2$ we get

$$E \approx m_\circ c^2 \left[1 + \frac{1}{2} \left(\frac{p}{m_\circ c} \right)^2 \right] \dots \dots \dots \dots . . (34)$$

Substitute equation (8) in equation (34) we get

$$E = m_\circ c^2 \left[1 + \frac{1}{2} \left(\frac{m_\circ v}{m_\circ c} \right)^2 \right]$$

$$E = m_\circ c^2 + \frac{1}{2} m_\circ v^2 \dots \dots \dots \dots \dots . . (35)$$

The first term is the rest mass of the particle and the second term is the classical kinetic energy.

DISCUSSION

Relativistic of energy must be conserved in all frames of reference. The relativistic momentum by Newton second law and relativistic of total energy by using Taylor series in (SR) are related according to the relation showing in equation (21) and equation (23) in the total energy by four vector in equation (29) equal to expression of energy in (SR) by using Newton second law and relativistic of total energy by using Taylor series in (SR) showing in equation (21), (23). Rewriting equation (23) to fined final classical kinetic energyin equation (35) .

CONCLUSION

They are many expression of energy by neglecting four vector energy momentum and (MRKE) and kinetic energy using wave function and relativistic momentum by Newton second law to find many expression of relativistic of energy.

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