An Active Path Finding Robot Based on Quantum Mechanics

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**Abstract:** By studying quantum state, quantum state spaces as well as multiple qubits of quantum mechanics, it is found that an active path finding robot can be designed on the basis of quantum mechanics. To begin with, quantum encoding, quantum logic gates, quantum logic circuit, quantum gate arrays and quantum parallelism are studied. Then a robot on the basis of quantum bits is designed. Finally, an active path finding algorithm on the basis of quantum fourier transforms and related pathing finding algorithm are found. Meanwhile, some examples and related analyses indicate that an active path finding robot on the basis of quantum mechanics is feasible.

**Keywords:** Path finding robot, active path finding, quantum mechanics, quantum searching algorithm.

## INTRODUCTION

### Related Work

First of all, it can be seen that different researchers put forward their own opinions on the related issues of path planning. Zhu, ZX created global path planning of wheeled robots using multi-objective memetic algorithms [1]. Montiel, O indicated path planning for mobile robots using bacterial potential field for avoiding static and dynamic obstacles [2]. Zhao, YM described calibration-based iterative learning control for path tracking of industrial robots [3]. MA proposed a mobile robot path planning using artificial bee colony and evolutionary programming [4], and Colombo implied a least restrictive supervisors for intersection collision avoidance with a scheduling approach [5].

Hossain, MA illustrated autonomous robot path planning in dynamic environment using a new optimization technique inspired by bacterial foraging technique [6]. Wang, MM considered trajectory planning of free-floating space robot using particle swarm optimization (PSO) [7].

In the same way, quantum mechanics research is also very popular recently. Wollman, EE implied quantum squeezing of motion in a mechanical resonator [8]. Bagrets, Dmitry talked about sachdev-ye-kitaev model as liouville quantum mechanics [9], Zurek, Eva indicated a predicting crystal structures and properties of matter under extreme conditions via quantum mechanics [10]. Quesne, Matthew G considered a quantum mechanics/molecular mechanics modeling of enzymatic processes with cavets and breakthroughs [11]. Some people even linked quantum mechanics to search algorithms and began to explore related research on quantum search algorithms. Barani, Fatemeh concerned application of binary quantum-inspired gravitational search algorithm in feature subset selection [12].

### Organization of the Article

Section 2 introduces some basic contents of quantum mechanics. Section 3 discusses how to design related robots based on quantum bits. In section 4, an active path finding algorithm based on quantum mechanics is put forward. In section 5, it contains the confirmations for virtual quantum laboratory. In section 6, instances and analyses are described. Section 7 is the result of the full text.

### The Base of Quantum Mechanics

#### Quantum State Spaces

The situation space of a quantum system, comprising of the spots, momentums, polarizations, spins, and etc. of the different particles, is imitated by a Hilbert space of wave operations. The details of these wave operations would not...
be looked at. For quantum computing dealing with finite quantum systems is enough and it is enough to think finite
dimensional complicated vector spaces with an inside product which are spanned by abstract wave operations like $|\psi\rangle$.

Kets like $|\psi\rangle$ indicate column vectors and are typically applied to made descriptions of quantum situations. The
matching bra, $\langle\psi|$ , indicates the conjugate transpose of $|\psi\rangle$. For instance, the orthonormal bases $\{|0\rangle, |1\rangle\}$ can be
described as $\{(1,0)^T, (0,1)^T\}$. Any complicated linear combining of $|\psi\rangle$ and $|\phi\rangle$, $\alpha|\psi\rangle + \beta|\phi\rangle$, can be written down $(\alpha,\beta)^T$.
Notice that the selection of the sequence of the basis vectors is random. For instance, standing for $|\psi\rangle$ as $(0,1)^T$ and $|\phi\rangle$ as $(1,0)^T$ will be well as long as this is operated consistently.

Making a Combination of $\langle\psi|$ and $|\phi\rangle$ as in $\langle\psi|\phi\rangle$, also written down as $\langle\psi|\phi\rangle$, indicates the inside product of
the two vectors. For example, since $|\psi\rangle$ is a unit vector is possessed $\langle\psi|\psi\rangle$, and since $|\psi\rangle$ and $|\phi\rangle$ are orthogonal there is
$\langle\psi|\phi\rangle$.

The notation $|\psi\rangle\langle\phi|$ is the outside product of $|\psi\rangle$ and $\langle\phi|$. For instance, $|\psi\rangle\langle\phi|$ is the change that maps $|\psi\rangle$ to $|\phi\rangle$
and $|\phi\rangle$ to $(0,0)^T$, since $|\psi\rangle\langle\phi| = |\psi\rangle\langle\psi| - |\phi\rangle\langle\phi| = |\psi\rangle - |\phi\rangle$.

Similarly, $|\psi\rangle\langle\phi|$ can be written down in matrix form, in which $|\psi\rangle = (1,0)^T$, $\langle\phi| = (0,1)^T$, $|\phi\rangle = (1,0)^T$, and $\langle\phi|$ = $(0,1)$. Then
$|\psi\rangle\langle\phi| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

This notation offers us a convenient means of specifying change on quantum situations in light of what takes
place in the basis vectors (see Section 4). For instance, the change that exchanges $|\psi\rangle$ and $|\phi\rangle$ is offered by the matrix.

In this paper the a little more intuitive notation is preferred

$\begin{pmatrix} |\psi\rangle \\ |\phi\rangle \end{pmatrix} \rightarrow
\begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$

which bluntly specifies the consequence of a change on the basis vectors.

Multiple Qubits

Assume a macroscopic physical matter separating apart and multiple pieces flying out in distinguishing
directions. The situation of this system can be expressed entirely by making descriptions of the situation of every
constituent pieces separately. An astonishing and unintuitive dimension of the situation space of an $n$-particle quantum
system is that the situation of the system could not always be expressed in light of the situation of its constituent pieces.
It is while checking systems of not only one qubit that one first has a glimpse of in which the computational force of
quantum computers can be from.

As we have seen, the situation of a qubit can be stood for by a vector in the two-dimensional complicated vector
space spanned through $|0\rangle$ and $|1\rangle$. In classic physics, the probable situations of a system of $n$ particles, of which
individual situations can be expressed through a vector inside a two-dimensional vector space, make up a vector space of
$2n$ dimensions. But in a quantum system the conclusive situation space is much huger; a system of $n$ qubits possess a
situation space of $2^n$ dimensions. It is this exponential growing of the situation space with the amount of particles which
indicates a probable exponential increasing of computation on quantum computers about classic computers.

Individual situation spaces of $n$ particles make combinations classically by the cartesian product. Quantum
situations, however, make combinations through the tensor product. The details upon natures of tensor products and their
description in light of vectors and matrices are offered here. Look transiently at differentiations between the cartesian
product and the tensor product which would be vital to understanding quantum computation.

Let $\Xi$ and $\Theta$ be 2 two-dimensional complicated vector spaces with bases $\{\xi_1, \xi_2\}$ and $\{\theta_1, \theta_2\}$ respectively. The cartesian product of the two spaces could take as its based part the union of the bases of its constituent spaces $\{\nu, \nu', \omega, \omega'\}$, Notice that the sequence of the basis was selected randomly. Particularly, the abstract of the situation space of multiple classic particles becomes linearly with the amount of particles, because $\dim(\lambda \times \varnothing) = \dim(\lambda) + \dim(\varnothing)$. The tensor product of $\Xi$ and $\Theta$ has basis $\{\xi_{1=[1,1], \xi_{2=[1,2], \omega=[1,2], \omega'=[2,2]}\}$. Notice that the sequence of the basis, again, is random. So the situation space for two qubits, every with basis that can be written down more concisely as $\{|\alpha\rangle, |\gamma\rangle\}$, has basis $\{|\alpha\rangle=|\nu\rangle=|\nu'\rangle=|\omega\rangle=|\omega'\rangle\}$ which can be written more concisely as $\{|\gamma\rangle=|\nu\rangle=|\nu'\rangle=|\omega\rangle=|\omega'\rangle\}$. More in general, $\{|\alpha\rangle\}$, has basis $\{|\gamma\rangle=|\nu\rangle=|\nu'\rangle=|\omega\rangle=|\omega'\rangle\}$. More in general, $\{|\alpha\rangle\}$ is written to indicate $\{|1\rangle^n_{n-1...0}\}$ in which $bi$ are the binary digits of the $x$.

For a 3-qubit system a basis is

$$\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$$

And generally an $n$-qubit system possesses $2^n$ basis vectors. It can be now seen the exponential growth of the situation space with the amount of quantum particles. The tensor product $\lambda \otimes \varnothing$ possesses dimension $\dim(\lambda) \times \dim(\varnothing)$.

The situation $\{|\gamma\rangle, |\nu\rangle\}$ is an instance of a quantum situation that could not be expressed in light of the situation of every components (qubits) separately. Alternatively, $a_1a_2b_1b_2$ cannot be found such that $(\xi_1|\gamma\rangle + \xi_2|\nu\rangle) \otimes (\xi_1|\gamma\rangle + \xi_2|\nu\rangle)$ is a basis since $(\xi_1|\gamma\rangle + \xi_2|\nu\rangle)$ is a basis. Just as in the photon polarization situation, if $\gamma, \nu \neq 0$, states that could not be decomposed in the means are named entangled situations. These situations stand for situations that have no classic counterpart and for which there is no intuition. These are also the situations that offer the exponential growing of quantum situation spaces with the number of particles.

Notice that it will need vast resources to imitate even a tiny quantum system on traditional computers. The development of quantum systems is exponentially quicker than their classic imitations. The reason for the potential power of quantum computers is the probability of using the quantum situation development as a computational mechanism.

Design a Robot by Quantum Bits

Quantum Encoding

A quantum bit is a unit vector in a two-dimensional complicated vector space for which a special basis, indicated by $\{|\eta\rangle\}$, has been made fixed. The orthonormal basis $|\gamma\rangle$ and $|\nu\rangle$ may coincide with the $|\gamma\rangle$ and $|\nu\rangle$ polarizations of a photon separately, or to the polarizations $|\gamma\rangle$ and $|\nu\rangle$. Or $|\gamma\rangle$ and $|\nu\rangle$ can coincide to the spin-up and spin-down situations of an electron. When discussing about qubits, and quantum computations generally, a fixed basis as to which all expressions are made has been selected in advance. Particularly, unless otherwise specified, all measuring would be made as to the standard bases for quantum computation, $\{|\gamma\rangle, |\nu\rangle\}$.

For the goals of quantum computation, the basis situations $|\gamma\rangle$ and $|\nu\rangle$ are taken to stand for the classic bit values 0 and 1 separately. However, unlike classic bits, qubits could be in a superposition of $|\gamma\rangle$ and $|\nu\rangle$ like $\sqrt{\alpha}|\alpha\rangle + \sqrt{\gamma}|\gamma\rangle$, in which $\alpha$ and $\gamma$ are complicated numbers like $|\alpha|^2 + |\gamma|^2 = 1$. Just as in the photon polarization situation, if a superposition like this is measured as to the basis $\{|\gamma\rangle, |\nu\rangle\}$, the possibility that the measured value is $|\gamma\rangle$ is $|\alpha|^2$ and the possibility that the value which is measured is $|\nu\rangle$ is $|\gamma|^2$.

Although a quantum bit can be put in $|\gamma|^2$, in infinitely lots of superposition situations, it is only probable to pick up a single classic bit’s worth of message from a simple quantum bit. The reason that less information can be gotten from a qubit than in a classic bit is that information can merely be gotten by measurement. While a qubit is measured, the measurement transforms the situation to one of the basis situations in the way observed in the photon polarization experiment. Since every measurement can lead to merely one of two situations, one of the fundamental vectors related to the given measuring device, so, just like in the classic situation, there are only two probable consequences. As measurement transforms the situation, one could not measure the situation of a qubit in two distinguishing bases. What’s more, as it can be seen in former Section, quantum situations could not be cloned, so it is impossible to measure a qubit in two means even not directly by, for example, copying the qubit and measuring the copy in a distinguishing basis from the initially.

Available online: http://scholarsmepub.com/sjet/
A \textit{n}k\textit{dimensional vector} The tensor production (\( \otimes \)) of a n-dimensional and a k-dimensional vector. Likewise, if \( \chi \) and \( \delta \) are changes on \textit{n}\textit{-dimensional and} \textit{k}-dimensional vectors separately, then \( \chi \otimes \delta \) is a change on \textit{n}k\textit{dimensional vectors.}

For our goals the following algebraic regulations are enough to count with tensor products. For matrices \( \chi, \delta, \kappa, \mu, \Gamma \), vectors \( u, x, y \), and scalars \( \alpha, \beta \), the following hold:

\[
(\chi \otimes \delta)(C \otimes D) = \chi C \otimes \delta D \\
(\chi \otimes \delta)(\alpha \otimes \beta) = \chi \alpha \otimes \delta \beta \\
\rho \otimes (\alpha + \beta) = \rho \otimes \alpha + \rho \otimes \beta \\
e \alpha \otimes \gamma \beta = e \gamma \alpha \otimes \beta
\]

\[
\left( \begin{array}{c}
\chi \\
\delta \\
\kappa \\
\mu
\end{array} \right) \otimes \Gamma = 
\left( \begin{array}{c}
\chi \otimes \Gamma \\
\delta \otimes \Gamma \\
\kappa \otimes \Gamma \\
\mu \otimes \Gamma
\end{array} \right)
\]

Which is specialized for scalars \( a, b, c, d \) to

\[
\left( \begin{array}{c}
e \\
\gamma \\
\zeta \\
\tau
\end{array} \right) \otimes \Gamma = 
\left( \begin{array}{c}
e \Gamma \\
\gamma \Gamma \\
\zeta \Gamma \\
\tau \Gamma
\end{array} \right).
\]

The conjugate transpose dispenses over tensor productions; that is,

\[
(\chi \otimes \delta)^* = \chi^* \otimes \delta^*
\]

If its conjugate transpose is its opposite:

\[
\Gamma^* \Gamma = \Pi, \chi \text{ matrix } \Gamma \text{ is unitary.}
\]

If and only if every matrice is unitary up to a constant, the tensor product of some matrices is unitary. Let \( \Gamma = \chi_1 \otimes \chi_2 \otimes \cdots \otimes \chi_n \). Next \( \Gamma \) is unitary if \( \chi_i^* \chi_i = k_i, \Pi \text{ and } \prod_i k_i = 1 \).

\[
\Gamma^* \Gamma = (\chi_1^* \otimes \chi_2^* \otimes \cdots \otimes \chi_n^*) (\chi_1 \otimes \chi_2 \otimes \cdots \otimes \chi_n) = \chi^* \chi \otimes \chi^* \chi \otimes \cdots \otimes \chi^* \chi
\]

\[
= 1 \Pi \otimes \cdots \otimes 1 \Pi = \Pi
\]

In which every \( \Pi \) indicates the identity matrix of proper dimension.

For instance, the distributive law permits computations of the form:

\[
\left( \varepsilon_0 |0 \rangle + \varepsilon_1 |1 \rangle \right) \otimes \left( \gamma_0 |0 \rangle + \gamma_1 |1 \rangle \right) = \left( \varepsilon_0 \gamma_0 |00 \rangle + \varepsilon_0 \gamma_1 |01 \rangle + \varepsilon_1 \gamma_0 |10 \rangle + \varepsilon_1 \gamma_1 |11 \rangle \right)
\]

\[
= \varepsilon_0 \gamma_0 (|00 \rangle + |01 \rangle) + \varepsilon_0 \gamma_1 (|01 \rangle + |11 \rangle) + \varepsilon_1 \gamma_0 (|00 \rangle + |10 \rangle) + \varepsilon_1 \gamma_1 (|10 \rangle + |11 \rangle)
\]

\[
= \varepsilon_0 \gamma_0 |00 \rangle + \varepsilon_0 \gamma_1 |01 \rangle + \varepsilon_1 \gamma_0 |10 \rangle + \varepsilon_1 \gamma_1 |11 \rangle
\]

\[
\text{Quantum Logic Circuit}
\]

The experiment in former Section manifests how measurement of a simplistic qubit reflects the quantum situation on to one of the basis situations related to the measuring device. The consequence of a measuring is probabilistic and the procedure of measurement transforms the situation to that measured.

Look at an instance of measuring in a two-qubit system. Any two-qubit situation can be described as \( |\varepsilon_0 \rangle + \gamma_0 |\varepsilon_1 \rangle + \zeta_0 |\tau_0 \rangle + \zeta_1 |\tau_1 \rangle \), in which \( \varepsilon, \gamma, \zeta, \tau \) are complicated numbers like \( |\varepsilon|^2 + |\gamma|^2 + |\zeta|^2 + |\tau|^2 = 1 \). Assume we hope to measure the first qubit as to the standard basis \( |\psi \rangle |\psi \rangle \). For convenience the state will be rewritten as follows:
\[
\varepsilon \left|00\right\rangle + \gamma \left|01\right\rangle + \zeta \left|10\right\rangle + \tau \left|11\right\rangle \\
= \left|0\right\rangle \otimes \left( \varepsilon \left|0\right\rangle + \gamma \left|1\right\rangle \right) + \left|1\right\rangle \otimes \left( \varepsilon \left|0\right\rangle + \tau \left|1\right\rangle \right)
\]

\[
= \rho \left|0\right\rangle \otimes \left( \varepsilon / \rho \left|0\right\rangle + \gamma / \rho \left|1\right\rangle \right) + \eta \left|1\right\rangle \otimes \left( \zeta / \eta \left|0\right\rangle + \tau / \eta \left|1\right\rangle \right)
\]

For \( \rho = \sqrt{\left|\varepsilon\right|^2 + \left|\gamma\right|^2} \) and \( \eta = \sqrt{\left|\zeta\right|^2 + \left|\tau\right|^2} \), the vectors \( \varepsilon / \rho \left|0\right\rangle + \gamma / \rho \left|1\right\rangle \) and \( \zeta / \eta \left|0\right\rangle + \tau / \eta \left|1\right\rangle \) are of unit length. Once the situation has been written again as above, as a tensor production of the bit being measured and a second vector of unit length, the probabilistic consequence of a measurement is simple to read. Measuring of the first bit would with possibility \( \rho^2 = \left|\varepsilon\right|^2 + \left|\gamma\right|^2 \) return \( \left|\varepsilon\right\rangle \), reflecting the situation to \( \left|\psi\right\rangle = \left( \varepsilon / \rho \left|0\right\rangle + \gamma / \rho \left|1\right\rangle \right) \), or with possibility \( \eta^2 = \left|\zeta\right|^2 + \left|\tau\right|^2 \) yield \( \left|\zeta\right\rangle \), reflecting the situation to \( \left|\psi\right\rangle = \left( \zeta / \eta \left|0\right\rangle + \tau / \eta \left|1\right\rangle \right) \). As \( \left|\psi\right\rangle = \left( \varepsilon / \rho \left|0\right\rangle + \gamma / \rho \left|1\right\rangle \right) \) and \( \left|\psi\right\rangle = \left( \zeta / \eta \left|0\right\rangle + \tau / \eta \left|1\right\rangle \right) \) are both unit vectors, no scaling is needed. Measuring the second bit operates likewise.

For the goals of quantum computation, multibit measuring can be regarded as a set of single-bit measurements in the norm basis. Other kinds of measurements are probable, like measuring if two qubits possess the same value without studying the real value of the two qubits. But such measuring is similar to unitary changes followed by a standard measuring of separate qubits, and so it is enough to look merely at standard measurements.

In the two-qubit instance, the situation space is a cartesian production of the subspace comprising of all situations of which first qubit is in the situation \( \left|\psi\right\rangle \) and the orthogonal subspace of situations of which first qubit is in the situation \( \left|\psi\right\rangle \). Any quantum situation could be written down as the total sum of two vectors, one in every subspaces. A measuring of \( k \) qubits in the standard basis possesses \( 2^k \) probable results \( m_i \). Every device measuring \( k \) qubits of an \( n \)-qubit system separates of the \( 2^n \)-dimensional situation space \( \Delta \) into a cartesian production of orthogonal subspaces \( \Delta_1, ... , \Delta_2^\eta \) with \( \Lambda = \Delta_1 \times \cdots \times \Delta_2^\eta \), like worth of the \( q \) qubits being measured is \( m_q \) and the situation after being measured is in space the space \( \Delta_q \) for several \( i \). The device arbitrarily selects one of the \( \Delta_q \)'s, with possibility the square of the amplitude of the constituent of \( \psi \) in \( \Delta_q \), and reflects the situation into that constituent, scaling to give length.

Similarly, the possibility that the consequence of the measurement is an offered worth is the total sum of the squares of the absolute worth of the amplitudes of all fundamental vectors in harmony with that worth of the measurement.

Measuring gives another means of considering about wriggled particles. Particles are not wriggled if the measuring of one possesses no effect on the other.

For example, the situation \( \frac{1}{\sqrt{2}} (\left|0\right\rangle \otimes \left|0\right\rangle) \) is entangled, since the possibility that the first bit is measured as \( \left|0\right\rangle \) is 1/2 if the second bit has not been measured. If, however, the second bit had been measured, the possibility that the first bit is measured as \( \left|0\right\rangle \) is either 1 or 0, relying on if the second bit was measured as \( \left|0\right\rangle \) or \( \left|1\right\rangle \) separately. Therefore, the probable consequence of measuring the first bit is transformed by a measuring of the second bit. On the other hand, the situation \( \frac{1}{\sqrt{2}} (\left|0\right\rangle \otimes \left|1\right\rangle) \) is not entangled: since \( \frac{1}{\sqrt{2}} (\left|0\right\rangle \otimes \left|0\right\rangle) = \frac{1}{\sqrt{2}} (\left|0\right\rangle \otimes \left|1\right\rangle) \). Any measuring of the first bit will produce \( \left|0\right\rangle \) despite if the second bit was measured. Likewise, the second bit has a fifty-fifty opportunity of being measured as \( \left|0\right\rangle \) in spite of ether the first bit was measured or not. Notice that entanglement, in the way that measuring of one particle has an influence on measuring of another particle, is equal to our former definition of entangled situations as situations that could not be written as a tensor production of separate states.

**An Active Path Finding Algorithm by Quantum Mechanics**

**Quantum Fourier transforms**

Generally map from the time rang to the frequency range. So Fourier changes map operations of period \( r \) to operations that possess nonzero values merely at multiples of the frequency \( \frac{r}{N} \). The discrete Fourier change (DFT) functions on \( N \) equivalently spaced samples in the interval \([0, 2\pi)\) for several \( N \) and outputs a operation of which range is
the integers between 0 and \( N - 1 \). The discrete Fourier change of a (sampled) operation of period \( \frac{N}{\omega} \) is an operation focused on next to multiples of \( \frac{N}{\omega} \). If the period \( \frac{N}{\omega} \) divides \( N \) similarly, the consequence is an operation that has nonzero values merely at multiples of \( \frac{N}{\omega} \). Otherwise, the consequence will be close to this behavior, and there would be nonzero terms at integers next to multiples of \( \frac{N}{\omega} \).

The Fast Fourier change (FFT) is an edition of DFT in which \( N \) is a force of 2. The quantum Fourier change (QFT) is a variation of the discrete Fourier change, which, like FFT, applies powers of 2. The quantum Fourier change functions on the amplitude of the quantum situation, through sending

\[
\sum_{\alpha} g(\alpha) \mid \alpha \rangle \rightarrow \sum_{\zeta} a(\zeta) \mid \zeta \rangle
\]

In which \( G(\zeta) \) is the discrete Fourier change of \( g(\alpha) \), and \( \alpha \) and \( \zeta \) both vary over the binary demonstrations for the integers between 0 and \( N - 1 \). If the situation were measured after the Fourier change was operated, the possibility that the consequence was \( \mid \zeta \rangle \) will be \( |G(\zeta)|^2 \). Notice that the quantum Fourier change does not output a operation the means the \( \Gamma f \) change does; no output emerges in an extra register.

Using the quantum Fourier change to a periodic operation \( g(\alpha) \) with period \( r \), it would be expected to end up with \( \sum_{\zeta} a(\zeta) \mid \zeta \rangle \), in which \( G(\zeta) \) is zero except at multiples of \( \frac{N}{\omega} \). Therefore, while the situation is measured, the consequence will be a multiple of \( \frac{N}{\omega} \), like \( \frac{N}{\omega} \). But as expressed above, the quantum Fourier change merely offers approximate consequences for periods which are not a force of two (i.e., do not divide \( N \)). But the huger the power of two applied as a basis for the change, the better the approximation. The quantum Fourier change \( U_{QFT} \) with basis \( N = 2^m \) is made definitions of by

\[
U_{QFT} : \mid \alpha \rangle \rightarrow \frac{1}{\sqrt{2^m}} \sum_{\zeta=0}^{2^m-1} e^{\frac{2\pi i \alpha \zeta}{2^m}} \mid \zeta \rangle
\]

For Shor’s algorithm to be a polynomial algorithm, the quantum Fourier change must be efficiently computable. Shor displays that the quantum Fourier change with basis \( 2^m \) can be constructed applying merely \( \frac{m(m+1)}{2} \) gates. The construction uses two types of gates. One is a gate to operate the familiar Hadamard change \( \Lambda \). We will indicte by \( \Lambda_j \) the Hadamard change used to the \( j \)th bit. The other type of gate operates 2-bit changes of the form

\[
\Lambda_{j,i} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\pi/2} \end{pmatrix}
\]

In which \( y_{k,j} = \pi/2^{j+k} \). This change acts on the \( r \)th and \( j \)th bits of a huger register. The quantum Fourier change is offered by

\[
\Lambda_0 \Lambda_0^{\dagger} \Lambda_{0,1} \Lambda_1^{\dagger} \cdots \Lambda_{m-3} \Lambda_{m-2}^{\dagger} \Lambda_{m-2, m-1} \Lambda_{m-1}^{\dagger}
\]

followed by a bit reversal chane.

**Path Finding of Shor’s algorithm**

The full steps of Shor’s algorithm are explained with a running instance in which \( M = 21 \) is factored.

Step-1 Quantum parallelism: Select an integer \( \epsilon \) randomly. If \( \epsilon \) is not relatively major to \( M \), a factor of \( M \), has been found. Otherwise use the rest of the algorithm.

Let \( m \) be like \( M^2 \leq 2^m < 2M^2 \). (This selection is made so that the approximating applied in Step 3 for operations of which period is not a force of 2 will be great sufficient for the rest of the algorithm to operate). Use quantum parallelism as expressed in Section 5.2 to compute \( f(\alpha) = \epsilon^\alpha \mod M \) for all integers from 0 to \( 2^m - 1 \). The operation is therefore encoded in the quantum situation.
\[
\frac{1}{\sqrt{m}} \sum_{a=0}^{2^m-1} \ket{a, \beta(a)}
\]  

(1)

**Instance.** Assume \( \varepsilon = 11 \) were arbitrarily selected. Since \( M^2 = 441 \leq 2^9 < 882 = 2M^2 \), there is \( m = 9 \). Therefore, a entire of 14 quantum bits, 9 for \( \alpha \) and 5 for \( f(\alpha) \), are needed to calculate the superposition of equation 1.

**Step 2** A situation of which amplitude has the same period as \( f \). The quantum Fourier change operates on the amplitude operation related to the input situation. To use the quantum Fourier change to get the period of \( f \), a situation is constructed of which amplitude operation possess the same period as \( f \).

To construct such a situation, measure the final \( \ket{\varepsilon} \) qubits of the situation of Eq. 1 which encodes \( f(\alpha) \). A arbitrary value \( \pi \) is gotten. The worth \( \pi \) is not of interest in itself; merely the influence the measuring has on our series of superpositions is of interest. This measuring reflects the situation space onto the subspace in harmony with the measured value, so the situation after measurement is

\[
e^{-\frac{\pi i}{\varepsilon}} \sum_{a} g(a) \ket{a, \rho}
\]

For some scale factor \( \kappa \) in which

\[
g(a) = \begin{cases} \frac{1}{\varepsilon} \ket{a} & \text{if } \varepsilon \\
0 & \text{otherwise}
\end{cases}
\]

Notice that the \( \alpha \)'s that in fact appear in the total sum, those with \( g(\alpha) \neq 0 \), distinguish from each other by multiples of the period; therefore \( g(\alpha) \) is the operation of what is looking for. If two successive \( \alpha \)'s can be measured in the sum, there would be the period. Unfortunately the ruls of quantum physics allows merely one measurement.

**Instance.** Assume that arbitrary measuring of the superposition of Eq. 1 invents 8. The situation after this measurement clearly displays the periodicity of \( f \).

**Step 3. Using a quantum Fourier change.** The \( \ket{\varepsilon} \) part of the situation would not be used, so it will not be written any longer. Apply the quantum Fourier change to the situation gotten in Step 2.

\[
\sum_{a} g(a) \ket{a} \rightarrow \sum_{\varepsilon} \alpha(\varepsilon) \ket{\varepsilon}
\]

Standard Fourier analysis shows us that while the period \( r \) of the operation \( g(\alpha) \) made definitions of in Step 2 is a power of 2, the consequence of the quantum Fourier change is

\[
\sum_{i} \varepsilon_j \ket{\sum_{i} \varepsilon_j}
\]

In which the amplitude is 0 other than at multiples of \( \frac{2m}{r} \). While the period \( r \) does not divide \( 2^m \), the change approximates the exact case, so a large amount of the amplitude is clinging to integers next to multiples of \( \frac{2m}{r} \).

**Instance.** Shor' alforithm shows the consequence of using the quantum Fourier change to the situation gotten in Step 2. Notice that Shor' alforithm is the graph of the quick Fourier change of the path finding operation. In this special instance the period of \( f \) does not divide \( 2^m \).

**Step 4. SElecting the period.** Measure the situation in the norm basis for quantum computation, and call the consequence \( \eta \). In the case in which the period happens to be a force of 2, so that the quantum Fourier change offers accurately multiples of \( \frac{2m}{r} \), the period is simple to extract. In this case, \( \eta_j \frac{2m}{r} \) for certain \( j \). A large amount of the time \( j \) and \( \bar{\epsilon} \) would be relatively major, where case reducing the fraction \( \frac{\eta_j}{\bar{\epsilon}} = \frac{1}{\bar{\epsilon}} \) to its lowest terms would invent a fraction of which denominator \( \bar{\epsilon} \) is the period \( \bar{\epsilon} \). The fact that generally the quantum Fourier change merely approximately offers
multiples of the scaled frequency complexes the picking out of the period from the measuring. While the period is not a
force of 2, a great guess for the period can be gotten using the continual fraction extention of $\frac{n}{2^m}$. This classic technique
is exspressed in

Example and Analysis

The main problem in establishing quantum computers is the requirement to separate the quantum situation. An
interaction of particles standing for qubits with the outside environment upsets the quantum situation and leads it to
decohere, or change in an unintended and usually nonunitary fashion.

While adding error correction algorithms to Shor’s algorithm diminishes the influence of decoherence, making it
again look probable that a system can be built on which Shor’s algorithm can be operated for large numbers.

At appearance, quantum mistake correcting is equivalent to classic error making corrections of codes because
redundant bits are applied to find and correct mistakes, but the state for quantum mistake correction is a little more
complex than in the classic case, since binary data but not quantum states is being dealt with.

Quantum error correction must reconstruct the accurate encoded quantum situation. Given the improbability of
cloning or copying the quantum situation, this reconstruction seems more difficult than in the classic case. But it turns
out that classic techniques could be modified to opearate for quantum systems.

As the static quantum systems have been looked at, which transform merely when it is measured. A quantum
system’s dynamics, while not being measured, are regulated by Schrodinger’s equation; the dynamics’ must take
situations to situations in a means that persists orthogonality. For a complicated vector space, linear changes that persist
orthogonality are unitary changes, made definitions of as follows. Any linear change on a complicated vector space could
be expressed by a matrix. Let $M^*$ indicate the conjugate transpose of the matrix $M$. A matrix $M$ is common (expresses a
unitary change) if $M M^* = I$. Any unitary change of a quantum situation space is a legitimate quantum change, and vice
versa. One can consider unitary changes as being rotations of a complicated vector space.

An apparent result of the fact that quantum changes are unitary is that they are revocable. Bennett, Fredkin, and Toffoli had already seen reversible editions of standard computing models displaying that all classic computations can be done revocably.

The use of simplistic quantum gates could be studied with two simplistic instances: dense coding and teleportation.

Dense coding enables one quantum bit as well as an EPR pair to encode and transfer two classic bits. Since EPR
pairs can be dispensed in advance, merely one qubit (particle) has to be physically transferred to convey two bits of
information. This consequence is surprising since, as was talked about in Section 3, merely one classic bit’s worth of
information could be picked from a qubit. Teleportation is the opposition of dense coding, because it exploits two classic
bits to transfer a single qubit. Teleportation is astonishing in terms of the no cloning rule of quantum mechanics, because
it enables the transfer of an unknown quantum situation.

The key to dense coding and teleportation is the exploitation of entangled particles. The first set up is the same
for both procedures, Alice and Bob hope to communicate. Each is sent one of the wriggled particles forming an EPR pair,

$$\psi_0 = \frac{1}{\sqrt{2}}(\mid 0 \rangle \cdot \mid 1 \rangle) .$$

For example, Alice is sent the inial particle, and Bob the second. Until a particle is transferred, merely Alice can
operate changes on her particle, and merely Bob can operation changes on his.

Alice. Alice accepts two classic bits, encoding the numbers 0 through 3. Relying on this number Alice operates
one of the changes { I, λ, o, Ω } on her qubit of the wriggled pair $\psi_0$. Changing just one bit of an wriggled pair indicates
operating the identity change on the other bit. The conclusive situation is shown in the table.

<table>
<thead>
<tr>
<th>Value</th>
<th>change</th>
<th>New situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\psi_0$</td>
<td>$\frac{1}{2^m} (\mid n \rangle \cdot \mid n \rangle)$</td>
</tr>
</tbody>
</table>
Alice then sends her qubit to Bob.

Bob applies a controlled-NOT to the two qubits of the entangled pair.

A table showing the initial situation, controlled-NOT operation, and second bit is provided. It is apparent that Bob could test the second qubit without disturbing the quantum situation. If the measurement comes back $|0\rangle$ then the encoded value was either 0 or 3, if the measurement comes back $|1\rangle$ then the encoded value was either 1 or 2.

Bob now applies $\Lambda$ to the first bit:

Lastly, Bob measures the conclusive bit, which permits him to differentiate between 0 and 3, and 1 and 2.

Alice has a qubit whose situation she doesn’t know. She wishes to send the situation of this qubit $\varphi$ to Bob through classic paths. Like dense coding, Alice and Bob each has one qubit of an entangled pair $\varphi = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$.

Alice can use the decoding procedure of dense coding to the qubit $\varphi$ to be transferred and her half of the entangled pair $\varphi = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$.

Where Alice regulates the first two bits and Bob controls the final one. Alice now uses $\kappa_{not} \otimes II$ and $A \otimes A \otimes II$ to this situation:
Alice tests the initial two qubits to get one of $|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$ with equivalent possibility. Depending on the result of the measurement, the quantum situation of Bob’s qubit is reflected to $|0\rangle$, $|1\rangle$, or $|\phi\rangle$ separately. Alice sends the consequence of her measuring as two classic bits to Bob.

While she measured it, Alice irretrievably changes the situation of her original qubit $\phi$, whose situation she is in the procedure of sending to Bob. This loss of the initial situation is the reason teleportation does not disobey the no cloning rule.

As Bob accepts the two classic bits from Alice he knows how the situation of his half of the entangled pair makes comparison to the initial situation of Alice’s qubit.

<table>
<thead>
<tr>
<th>Bits received</th>
<th>situation</th>
<th>decoding</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>$</td>
<td>\phi\rangle$</td>
</tr>
<tr>
<td>01</td>
<td>$</td>
<td>1\rangle$</td>
</tr>
<tr>
<td>10</td>
<td>$</td>
<td>0\rangle$</td>
</tr>
<tr>
<td>11</td>
<td>$</td>
<td>1\rangle$</td>
</tr>
</tbody>
</table>

Bob will reconstruct the initial situation of Alice’s qubit, $\phi$, by using the proper decoding change to his aspect of the entangled pair. Notice that this is the encoding process of dense coding.

**CONCLUSIONS**

Through our research, a robot is designed by studying quantum encoding, quantum logic gates, quantum logic circuit, quantum gate arrays and quantum parallelism. Then, an active path finding algorithm based on quantum fourier transforms and related pathing finding algorithm are found. Finally, through related examples and analysis, it is found out that an active path finding robot based on quantum mechanics is feasible.

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