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Flexural Analysis of Clamped Thin Rectangular Isotropic Plates Using Galerkin Variational Method

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Abstract: The flexure of a clamped uniformly-loaded thin isotropic rectangular plate is herein analysed. The deflected surface was approximated using a grid work of beams. Coordinate polynomial deflection function satisfying the geometric boundary conditions was derived. Furthermore, different approximations of the derived polynomial function were developed for the clamped rectangular plate corresponding to the first, second and third approximations. The unknown deflection coefficients of the different deflection functions were obtained using Galerkin method, for different aspect ratios ranging from 1.0 to 2.0 for the different approximations. The numerical values of the computed coefficients were compared with the results from previous works and the degree and pattern of convergence observed. The convergence to the results of the classical solution increased as the number of approximations increased from first to third approximation such that the average percentage difference came to 1.98 %, for maximum deflection and 1.96 % for the maximum span moment of the principal axis in the third approximation. This clearly shows that the present study closely compares with the results of classical solution.

Keywords: Coordinate polynomial, Deflection function, Boundary conditions, Clamped plate, Rectangular plate, Uniform load, Galerkin method.

Nomenclature

A	surface area of plate
a	primary axis of plate
b	secondary axis of plate
D	flexural rigidity
E	young modulus
h	plate thickness
M	maximum bending moment
q	external load
w (.) $\overline{w}$ (.)	deflection function, trial function
ν	poisson's ratio

INTRODUCTION

The various applications of rectangular plates in engineering and construction cannot be overemphasized. As a result, the problem of a rectangular plate clamped on all four edges subjected to a uniformly distributed load is of great importance and some papers have been devoted to this.

However, because the exact solution of the problem of a rectangular plate is possible only for simply supported plates [1], obtaining accurate results for fixed rectangular thin plates appear to be difficult. Many authors have evaluated the deflections and moments of rectangular plates with various supports using different methods [2-6]. Most analysts have used the approximation techniques of Rayleigh-Ritz, Ritz, Improved Kantorovich, Superposition etc.

The accuracy of the solution of the uniformly loaded rectangular plate with clamped edges depends on the suitability of the deflection function chosen. Consequently, to choose the deflection function that best approximates the deflected middle surface of the loaded plate requires serious attention and care. [7], employed deflection functions comprising trigonometric and hyperbolic series for the problem of an isotropic rectangular plate with four edges clamped. [8], investigated the bending of rectangular plates using trigonometric functions in a Semi-numerical method. [9, 10], used a one-term polynomial deflection function to solve for the maximum deflection function in Ritz method. On the other hand [11], extended previous works done by [10] by using polynomial functions in Galerkin method to investigate the bending of clamped orthotropic rectangular plates. However, they did not show how these displacement functions could be derived for further plate analysis. Most analysts adopt trigonometric functions as deflection functions, others hyperbolic and polynomial functions or a combination of these. The advantage of trigonometric functions is the continuous nature of their derivatives. But for complex geometry and loading, it is difficult to choose a deflection function using the trigonometric or hyperbolic functions. Even so, most polynomial functions lack systematic derivation thereby lending themselves to guesswork.

This paper, therefore, analyzes the maximum deflections and span moments of a clamped thin rectangular isotropic plate under uniformly distributed load. A systematic and comprehensive method is presented for the Galerkin variational solution of the clamped rectangular plate. First, multi-term polynomial deflection functions corresponding to the first, second and third approximations are derived. The Galerkin method is used to analyze the numerical values of the unknown deflection coefficients and their corresponding mid-span moment coefficients for the first, second and third approximations. These results are compared with the classical solution of [12] and their degree and pattern of convergence observed.

METHODOLOGY

The deflected middle surface of a uniformly loaded rectangular plate as shown in Fig-1 can be approximated using a grid work of beams. The plate has all the edges clamped.

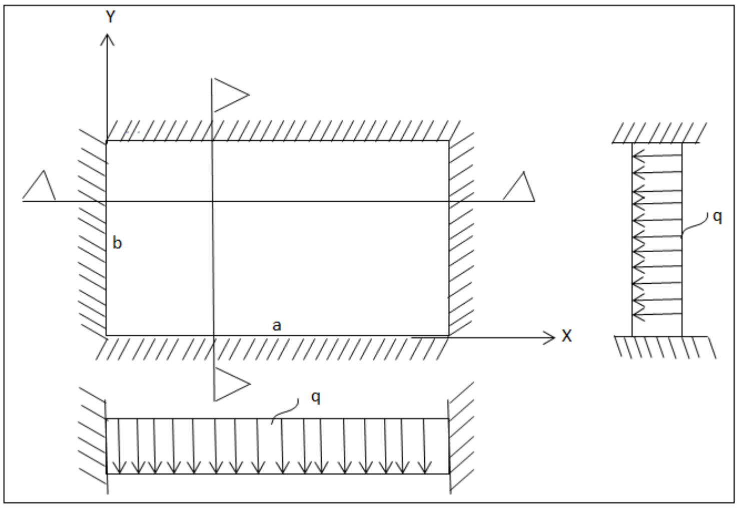

Fig-1: A rectangular plate with all edges clamped subjected to a uniformly distributed load

Since a plate bears its loads in two directions, two beams each running parallel to the x and y axes and perpendicular to each other are idealized. The appropriate deflected surface must satisfy at least two prescribed conditions at each boundary point. For the clamped plate in Fig.1, the boundary conditions are:

$$w = \frac{dw}{dx} = 0 \quad \text{at } x=0,\ 1 \qquad (1a)$$

$$w = \frac{dw}{dy} = 0 \quad \text{at } y=0,\ 1 \qquad (1b)$$

For the clamped plate in Fig.1, it is assumed that the deflection function w can be represented in the form of polynomials as follows:

$$W(x) = \sum_{m=0}^{\infty} A_m x^m \qquad (2a)$$

$$W(y) = \sum_{n=0}^{\infty} B_n y^n \qquad (2b)$$

Where x^m and y^n denote complete sets of independent continuous functions suitable for the representation of the deflected surface. Coefficients A_m and B_n are determined from the prescribed boundary conditions of the plate while m and n are determined by the type of loading on the plate.

First approximation

For the primary axis, if we take moment at any point along the element, equation (2a) gives:

$$W(x) = A_o + A_1X + A_2X^2 + A_3X^3 + A_4X^4 \qquad (3)$$

Applying the boundary conditions in equation (1a) in equation (3) gives

$$W(x) = A_4(X^2 - 2X^3 + X^4) \qquad (4)$$

Similarly, equation (2b) gives:

$$W(y) = B_0 + B_1Y + B_2Y^2 + B_3Y^3 + B_4Y^4 \qquad (5)$$

Applying the boundary conditions in equation (1b) in equation (5) gives

$$W(y) = B_4(Y^2 - 2Y^3 + Y^4) \qquad (6)$$

Therefore, the coordinate polynomial displacement function for clamped rectangular plate based on static deflection configuration is:

$$W(x, y) = C(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) \qquad (7)$$

Where C is an unknown deflection coefficient to be determined, and it is defined as given in equation (8).

$$C = A_4 * B_4 \qquad (8)$$

Second approximation

Now we derive a three-term deflection function from equation (7) as follows:

$$W(x, y) = C(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + (X^2 + Y^2)[C(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)]$$

$$W(x, y) = C_1(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + C_2(X^4 - 2X^5 + X^6)(Y^2 - 2Y^3 + Y^4) + C_3(X^2 - 2X^3 + X^4)(Y^4 - 2Y^5 + Y^6) \qquad (9)$$

Third approximation

Here we derive a six-term deflection function from equation (7) as follows:

$$W(x, y) = C(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + (X^2 + Y^2)[C(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)] + (X^2Y^2 + X^4 + Y^4)[C(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)]$$

$$W(x, y) = C_1(X^2 - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + C_2(X^4 - 2X^5 + X^6)(Y^2 - 2Y^3 + Y^4) + C_3(X^2 - 2X^3 + X^4)(Y^4 - 2Y^5 + Y^6) + C_4(X^4 - 2X^5 + X^6)(Y^4 - 2Y^5 + Y^6) + C_5(X^6 - 2X^7 + X^8)(Y^2 - 2Y^3 + Y^4) + C_6(X^2 - 2X^3 + X^4)(Y^6 - 2Y^7 + Y^8) \qquad (10)$$

Formulation of Galerkin functional

According to the classic theory of plate bending, the governing differential equation for isotropic thin plate is:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad (11)$$

Where w is the small deflection of the plate middle surface, q is the intensity of the external load, and $D = Eh^3/12(1 - \nu^2)$ is the constant flexural rigidity of the plate.

Furthermore, the maximum span moments acting on the plate can be calculated from the displacement function w through the following relations:

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right] \qquad (12)$$

Similarly,

$$M_y = -D\left[\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] \tag{13}$$

The Galerkin formulation of the plate bending problem for an isotropic rectangular plate is given in Cartesian coordinates as follows:

$$\iint_A \left(D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D\frac{\partial^4 w}{\partial y^4} - q\right)\bar{w}_2(x,y)dxdy = 0$$

$$\iint_A \left(D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D\frac{\partial^4 w}{\partial y^4} - q\right)\bar{w}_2(x,y)dxdy = 0$$

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$$\iint_A \left(D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D\frac{\partial^4 w}{\partial y^4} - q\right)\bar{w}_N(x,y)dxdy = 0 \tag{14}$$

The integrals are evaluated over the entire surface area A of the plate and $\bar{w}_{1....N}(x,y)$ are the linearly independent displacement functions that satisfy all the prescribed boundary conditions but not necessarily equation (11).

Maximum deflections and maximum moments of laterally loaded rectangular plates

For the deflection parameters of the rectangular plate due to the first approximation deflection functions, equation (7) is substituted into equation (14) and the resulting linear equation is solved for the unknown coefficient C. Once C is known, the deflections can be determined from equation (7) while the maximum span moments can be determined from equations (12) and (13) by differentiating accordingly, the determined displacement functions.

For the second approximation deflection parameters, equation (9) is substituted into equation (14) and solving the resulting 3 x 3 equation, the coefficients C_1, C_2, and C_3 are evaluated. Afterwards, the determined deflection coefficients are substituted into equation (9) to give the deflection of the plate. The moment coefficients can be worked out from equations (12) and (13) respectively, for the x and y axes using the deflection values already obtained.

For the third approximation parameters, equation (10) is put into equation (14) and solving the ensuing 6 x 6 equation, the coefficients C_1, C_2, C_3, C_4, C_5 and C_6 are calculated. Once calculated, the determined deflection coefficients are substituted into equation (10) to get the deflection of the plate. The moment coefficients are evaluated by putting the values of the determined deflections in equations (12) and (13) for the maximum moments in x and y axes respectively.

Numerical example

A uniformly loaded clamped rectangular plate as shown in Fig.1 is considered. The problem is solved for the first, second and third approximations and the obtained results are compared with the classical solution. The value of the poisson's ratio is taken as ν = 0.3. The numerical values of the deflection can be calculated if the deflection coefficients are known for all values of the aspect ratio (p = b/a). For the first approximation for instance, by substituting equation (7) into equation (14), we have:

$$a_{11} = \frac{D}{a^4}\iint_A\left[\frac{\partial^4\bar{w}_1}{\partial X^4} + 2\frac{\partial^4\bar{w}_1}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4\bar{w}_1}{\partial Y^4}\frac{1}{P^4}\right]\bar{w}_1(X,Y)dXdY \tag{15}$$

For the external load however, we have:

$$b_1 = \iint_A q\bar{w}_1(x,y)dxdy \tag{16}$$

Integrating equations (15) and (16) over the entire area A of the plate and substituting same into equation (14) gives:

$$C = \frac{b_1}{a_{11}} = \alpha\frac{qa^4}{D} \tag{17}$$

The determined coefficient C is put back into equation (7) to get the deflection of the plate. The corresponding moment coefficients in x and y axes are evaluated from equations (12) and (13) respectively.

The second approximation gives:

$$a_{11} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_1}{\partial X^4} + 2\frac{\partial^4 \overline{w}_1}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_1}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_1(X,Y)dXdY \qquad (18)$$

$$a_{12} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_1}{\partial X^4} + 2\frac{\partial^4 \overline{w}_1}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_1}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_2(X,Y)dXdY \qquad (19)$$

$$a_{13} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_1}{\partial X^4} + 2\frac{\partial^4 \overline{w}_1}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_1}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_3(X,Y)dXdY \qquad (20)$$

For the external load however, we have:

$$b_1 = \iint_A q\overline{w}_1(X,Y)dXdY \qquad (21)$$

$$a_{1,1}C_1 + a_{1,2}C_2 + a_{1,3}C_3 = \frac{b_1}{D}a^4 \qquad (22)$$

For the second term deflection parameters, we have:

$$a_{21} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_2}{\partial X^4} + 2\frac{\partial^4 \overline{w}_2}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_2}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_1(X,Y)dXdY \qquad (23)$$

$$a_{22} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_2}{\partial X^4} + 2\frac{\partial^4 \overline{w}_2}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_2}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_2(X,Y)dXdY \qquad (24)$$

$$a_{23} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_2}{\partial X^4} + 2\frac{\partial^4 \overline{w}_2}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_2}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_3(X,Y)dXdY \qquad (25)$$

For the external load however, we have:

$$b_2 = \iint_A q\overline{w}_2(X,Y)dXdY \qquad (26)$$

Hence,

$$a_{2,1}C_1 + a_{2,2}C_2 + a_{2,3}C_3 = \frac{b_2}{D}a^4 \qquad (27)$$

For the third term deflection parameters, we have:

$$a_{31} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_3}{\partial X^4} + 2\frac{\partial^4 \overline{w}_3}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_3}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_1(X,Y)dXdY \qquad (28)$$

$$a_{32} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_3}{\partial X^4} + 2\frac{\partial^4 \overline{w}_3}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_3}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_2(X,Y)dXdY \qquad (29)$$

$$a_{33} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4 \overline{w}_3}{\partial X^4} + 2\frac{\partial^4 \overline{w}_3}{\partial X^2\,\partial Y^2}\frac{1}{P^2} + \frac{\partial^4 \overline{w}_3}{\partial Y^4}\frac{1}{P^4}\right]\overline{w}_3(X,Y)dXdY \qquad (27)$$

For the external load however, we have:

$$b_3 = \iint_A q\overline{w}_3(X,Y)dXdY \qquad (28)$$

Hence,

$$a_{3,1}C_1 + a_{3,2}C_2 + a_{3,3}C_3 = \frac{b_3}{D}a^4 \qquad (29)$$

Second Approximation

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \frac{qa^4}{D}$$

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \frac{a^4}{D} \qquad (30)$$

The determined coefficients C_1, C_2 and C_3 are put back into equation (9) to get the deflection of the plate. The corresponding moment coefficients in x and y axes are evaluated from equations (12) and (13) respectively.

Following the same procedure above, the 6 X 6 simultaneous equations for the third approximation is solved for the unknown coefficients C_1, C_2, C_3, C_4, C_5 and C_6. The determined coefficients are put back into equation (10) to get the deflection of the plate. The corresponding moment coefficients in x and y axes are evaluated from equations (12) and (13) respectively.

RESULTS AND DISCUSSION

Deflection

The determined deflection coefficients for aspect ratios $1.0 \leq b/a \geq 2.0$ of the rectangular plate are shown in Table 1 for the first, second and third approximations. It is observed that the pattern of convergence is such that the coefficients converge as the approximation is increased from first, through second, to third approximation. The first and second approximation values are all upper-bounded and their percentage differences with the classical solution almost increasing from aspect ratio 1.0 and peaking at 2.0. The third approximation is lower-bounded from aspect ratio 1.0 to 1.6. The average percentage differences between the classical solution and present study for the first, second and third approximations are 7.25%, 7.08% and 1.98% respectively. This indicates that the deflection coefficient values converged from the first approximation to the third approximation.

Table-1: Mid-span (X =0.5, Y =0.5) Deflection Coefficient Values, α, for Clamped Rectangular Plate at Varying Aspect ratios ($W_{max} = (\alpha qa^2/D)$)

Aspect ratio, P	Present Study			Classical Solution
	W_1	W_2	W_3	W
	First Approximation	Second Approximation	Third Approximation	Timoshenko and Woinowsky-Krieger (1970)
1.0	0.00133(5.56%)	0.00132(4.92%)	0.00120(-5.10%)	0.00126
1.1	0.00159(6.00%)	0.00158(5.14%)	0.00144(-4.00%)	0.00150
1.2	0.00182(5.81%)	0.00181(5.15%)	0.00166(-3.26%)	0.00172
1.3	0.00202(5.76%)	0.00201(5.37%)	0.00186(-2.53%)	0.00191
1.4	0.00220(6.28%)	0.00219(5.74%)	0.00203(-1.85%)	0.00207
1.5	0.00235(6.82%)	0.00234(6.33%)	0.00217(-1.16%)	0.00220
1.6	0.00248(7.83%)	0.00247(7.26%)	0.00229(-0.33%)	0.00230
1.7	0.00259(8.82%)	0.00258(8.20%)	0.00239(0.35%)	0.00238
1.8	0.00269(9.80%)	0.00267(8.86%)	0.00246(0.60%)	0.00245
1.9	0.00277(11.24%)	0.00275(10.24%)	0.00252(1.39%)	0.00249
2.0	0.00284(11.81%)	0.00281(10.70%)	0.00257(1.21%)	0.00254

Short Span Moment Coefficients

Table 2 shows the short span moment coefficient values at the center for aspect ratios $1.0 \leq b/a \geq 2.0$. The moment coefficient values followed the same pattern as the deflection coefficients, converging from the first, through second, to the third approximation. The approximations have average percentage differences of 17.57%, 15.62% and 1.96% respectively when compared with the classical solution. The percentage difference is shown to have increased when compared to that of deflection. This is expected as the deflection coefficient values are evaluated to a higher degree of accuracy than the moment coefficient values. This is due to the fact that the stress couples are proportional to the second derivatives of the deflection functions.

Table-2: Short Span Moment Coefficient Values, β_x, at Mid-Span (x =0.5, y =0.5) for Clamped Rectangular Plate at Varying Aspect Ratio ($(M_x)_{max}) = qa^2\beta_x$)

Aspect ratio, P	Present Study			Classical Solution
	Mx1	Mx2	Mx3	Mx
	First Approximation	Second Approximation	Third Approximation	Timoshenko and Woinowsky-Krieger (1970)
1	0.02765(19.70%)	0.02708(17.25%)	0.02322(0.54%)	0.02310
1.1	0.03167(19.96%)	0.03107(17.67%)	0.02701(2.32%)	0.02640
1.2	0.03517(17.63%)	0.03454(15.52%)	0.03035(1.52%)	0.02990
1.3	0.03814(16.64%)	0.03750(14.67%)	0.03320(1.53%)	0.03270
1.4	0.04064(16.45%)	0.03997(14.53%)	0.03556(1.89%)	0.03490
1.5	0.04270(16.03%)	0.04202(14.19%)	0.03747(1.83%)	0.03680
1.6	0.04441(16.56%)	0.04372(14.74%)	0.03899(2.33%)	0.03810
1.7	0.04582(16.89%)	0.04512(15.10%)	0.04016(2.45%)	0.03920
1.8	0.04699(17.18%)	0.04628(15.41%)	0.04105(2.37%)	0.04010
1.9	0.04796(17.84%)	0.04724(16.08%)	0.04170(2.47%)	0.04070
2	0.04877(18.37%)	0.04805(16.63%)	0.04216(2.34%)	0.04120

Long Span Moment Coefficients

Table 3 shows the long span moment coefficient values at the center for aspect ratios $1.0 \leq b/a \geq 2.0$. These coefficient values followed the same pattern as the preceding coefficients, converging from the first, through second, to the third approximation. The approximations have average percentage differences of 38.74%, 34.93% and 8.46% respectively when compared with the classical solution. As would be expected, this loss of accuracy is down to the stress couples being proportional to the second derivatives of the deflection function.

Table-3: Long Span Moment Coefficient Values, β_y, at Mid-Span (x =0.5, y =0.5) for Clamped Rectangular Plate at Varying Aspect Ratio ($(M_y)_{max} = qa^2\beta_y$)

Aspect ratio, P	Present Study			Classical Solution
	My1	My2	My3	My
	First Approximation	Second Approximation	Third Approximation	Timoshenko and Woinowsky-Krieger (1970)
1	0.02765(19.70%)	0.02708(17.25%)	0.02322(0.54%)	0.02310
1.1	0.02858(23.72%)	0.02795(21.00%)	0.02390(3.48%)	0.02310
1.2	0.02894(26.93%)	0.02825(23.92%)	0.02401(5.31%)	0.02280
1.3	0.02889(30.14%)	0.02815(26.81%)	0.02369(6.71%)	0.02220
1.4	0.02855(34.67%)	0.02778(31.06%)	0.02307(8.83%)	0.02120
1.5	0.02805(38.18%)	0.02726(34.28%)	0.02226(9.68%)	0.02030
1.6	0.02745(42.23%)	0.02665(38.09%)	0.02135(10.65%)	0.01930
1.7	0.02682(47.36%)	0.02601(42.92%)	0.02040(12.10%)	0.01820
1.8	0.02618(50.46%)	0.02537(45.83%)	0.01945(11.78%)	0.01740
1.9	0.02555(54.85%)	0.02476(50.05%)	0.01853(12.29%)	0.01650
2	0.02495(57.91%)	0.02417(53.00%)	0.01765(11.73%)	0.01580

*Values in bracket are the percentage difference between the present study and the classical solution.

CONCLUSIONS

The results of calculations for maximum deflection and maximum span moment coefficients for aspect ratios $1.0 \leq b/a \geq 2.0$ have been computed using derived multi-term coordinate polynomial deflection functions in Galerkin method. The results have been compared with the results in literature and they compare closely with the classical solution. It is observed that the convergence to the classical solution increased as the number of approximations increased from the first, through second, to the third approximation, giving an average percentage difference of less than 2% for the third approximation, for the maximum deflection and maximum moment for the short span. These results show that for better accuracy, the number of approximations should be increased to the six-term deflection functional to better represent the deflected middle surface of the plate. Clearly, it is hereby concluded that the six-term coordinate polynomial deflection function of the third approximation herein derived, is suitable for the analysis of clamped rectangular plate problems. Nevertheless, since the first and second approximation values are in upper-bound, they can be used for practical purposes and preliminary studies.

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